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Multi-Objective Optimization of Lunar Navigation Satellite System using Genetic Algorithm

Rameez A. Malik[a]*, Yang Yang[a]

[a] *School of Mechanical and Manufacturing Engineering, University of New South Wales, 2052, Sydney Australia*
* Rameez A. Malik

Abstract

The future Lunar missions (orbiters, landers and rovers) planned by numerous national space agencies require reliable navigation and timing services. Recent studies regarding reception of Earth-based navigation system signals in cislunar space have shown promising prospects. However, they have also identified view geometry constrains that limits coverage and positional accuracy. A dedicated Lunar Navigation Satellite System (LNSS) will be required for real-time positioning to guarantee the safety of mission during all critical operational phases, e.g. descent and landing and surface operations, etc.

Previous research in LNSS design in Low Lunar Orbits (LLO) is mostly conducted with analytical geometric analysis of limited configurations, e.g. frozen orbit conditions with large number of satellites. Moreover, they consider few performance indicators and lack orbital maintenance analysis. The prior work in LNSS optimization framework focuses on global lunar coverage evaluation. Moreover, they incorporate analytical solution of station keeping ΔV budget derived from mean orbital elements, which is a low precision approach. This paper explores LNSS constellation design for real-time positioning in the cislunar domain and targeted lunar regions, e.g. south pole by complying with accuracy criteria outlined by the international space organizations. The south pole is selected due its strategical importance and natural resource availability. Meanwhile, secular and periodic perturbations are computed for more realistic investigation of ΔV station-keeping budget with a low thrust manoeuvre model.

The problem is addressed by opting multi-Objective Non-Dominated Sorting Genetic Algorithm II (NSGA-II) to optimize the space mission design as a function of mission cost, navigation performance and orbital stability. The algorithm is integrated with Lunar High Precision Orbit Propagation (LHPOP) model with real time perturbation forces. The objective function is derived from Figure of Merits (FOMs) including Positional Dilution of Precision (PDOP), Horizontal Dilution of Precision (HDOP), satellite count and station keeping ΔV. The decision variables include set of six Keplerian orbital elements, representing constellation configuration. The aim is to find pareto-optimal solutions representing best trade-offs between the objective function variables. For comparative analysis, NSGA-II is compared with a pattern-based Pareto search algorithm. The results indicate that NSGA-II is computationally efficient and produced a diverse range of solutions across the design space. In contrast, the Pareto search method is computationally expensive and primarily identifies concentrated regions of the design space that yield highly accurate FOMs, but at the expense of diversity and broader exploration.

Keywords: Navigation, Optimization, Pareto front, station-keeping maneuver and south pole

Nomenclature

ω	Argument of periapsis
ΔV	Delta-V
e	Eccentricity
$HDOP_{3\sigma\text{-}mean}$	Mean of spatio-temporal HDOP
i	Inclination angle
M	Mean anomaly
N_p	Number of planes
N_s	Number of satellites per plane
$PDOP_{3\sigma\text{-}mean}$	Mean of spatio-temporal PDOP
Ω	Right ascension of the ascending node
a	Semi-major axis

Acronyms/Abbreviations

DOP	Dilution of Precision
DRO	Distant Retrograde Orbits
ELFO	Elliptical Lunar Frozen Orbits
FOMs	Figure of Merits
GVEs	Gauss's Variational Equations
GDOP	Geometric Dilution of Precision
GA	Genetic Algorithm
GPS	Global Positioning System
HDOP	Horizontal Dilution of Precision
LPO	Liberation Points Orbits
LVLH	Local Vertical Local Horizontal
LLO	Low Lunar Orbit
LFOs	Lunar Frozen Orbits
LHPOP	Lunar High Precision Orbit Propagation
LNSS	Lunar Navigation Satellite System
MCI	Moon Centred Inertial
MCMF	Moon Centred Moon Fixed
MOO	Multi Objective Optimization
MOEA	Multi-Objective Evolutionary Algorithm
NRHO	Near-Rectilinear Halo Orbit

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NSGA-II Non-Dominated Sorting Genetic Algorithm II
PDOP Positional Dilution of Precision
RTN Radial-Tangential-Normal

1. Introduction

Lunar exploration has once again emerged as a central priority for global space agencies, motivated by the objective of establishing a sustainable human and robotic presence beyond Earth. Upcoming programs, including Artemis and various international initiatives, envision long-duration operations supported by orbiters, landers, and surface habitats for resource utilization [1]. These endeavours necessitate precise and continuous navigation capabilities to ensure mission safety and operational efficiency. Within this framework, the establishment of a dedicated Lunar Navigation Satellite System (LNSS) constitutes a critical enabler, delivering reliable positioning, guidance, and timing services for both crewed and uncrewed missions in the lunar vicinity. Achieving such capabilities requires the extension of Earth-based global navigation paradigms, such as the Global Positioning System (GPS), into cislunar space to provide high-accuracy positioning and navigation [2].

In recent years, the concept of an autonomous lunar navigation network has become a focal point within the global aerospace research community, with institutions investing heavily in its feasibility. Extensive research has analytically explored a variety of orbital options for lunar navigation satellites, including Low Lunar Orbits (LLO), Liberation Points Orbits (LPO), Near-Rectilinear Halo Orbits (NRHO), Distant Retrograde Orbits (DRO), and Elliptical Lunar Frozen Orbits (ELFO) [3, 4, 5]. These studies typically evaluate configurations based on individual performance factors such as coverage, visibility, and Dilution of Precision (DOP), but often lack a holistic assessment that integrates both performance metrics with other mission design constraints. For instance, studies relying on distant orbits like DRO and NRHO overlooked practical limitations in the LNSS to lunar user communication link, such as restricted onboard power due to greater Earth–Moon distances. Similarly, mission cost is usually expressed in terms of the required number of satellites, yet most analyses do not incorporate perturbation effects, which directly influence constellation station-keeping demands and the associated ΔV budget.

Relying solely on geometric analysis is insufficient for selecting an optimal constellation, particularly in multi-dimensional mission design problems. Such mission designs demand advanced optimization methods capable of addressing dynamic design parameters, including orbital configurations and mission objectives. Several prior studies have also applied numerical optimization techniques to lunar navigation system design. For example, Pereira et al. [6] examined the architecture of Lunar Frozen Orbits (LFOs) using the Borg Multi-Objective Evolutionary Algorithm (MOEA), where the fitness function incorporated Geometric Dilution of Precision (GDOP), space segment cost, and constellation ΔV requirements. While the study offered valuable preliminary insights, its scope was limited to LFO based configurations, thereby restricting the solution space to a narrow orbital regime. Additionally, ΔV manoeuvres were computed analytically within the Local Vertical Local Horizontal (LVLH) frame using mean Keplerian orbital elements. This limitation was later addressed by Arcia et al. [7], who applied a Multi-Objective Optimization (MOO) framework across a broader range of orbital parameters. Their approach considered PDOP, HDOP and constellation size as performance metrics, achieving a 44% improvement in PDOP compared to Pereira et al. with the same number of satellites. However, station-keeping ΔV was still simplistic with many assumptions.

This research is centred on astrodynamical considerations such as constellation architecture and comprehensive ΔV budgeting, with the overarching goal of developing a framework that balances cost efficiency and operational effectiveness. The selection of orbit type and constellation configuration must be driven by mission objectives and design constraints, while navigation performance is evaluated through parameters such as satellite visibility, DOP and positioning accuracy. Current global initiatives in cislunar positioning aim for navigation accuracy better than 50 meters within this decade and achieving this benchmark serves as a key objective of the present study [7]. To address these goals, MOO approach is employed, recognizing that, unlike Earth-based navigation constellations, lunar systems face unique challenges arising from the large Earth-Moon distance, complex multibody perturbations from the Earth and Sun. Therefore, a comprehensive analysis is performed that integrates navigation service requirements in lunar south pole with constellation geometry and orbital stability. This targeted coverage approach is mainly employed to minimize the constellation size without compromising the coverage of Moon's most interested and strategically important location. Similarly, the paper places strong emphasis on the station-keeping ΔV budget, evaluated through a continuous manoeuvre model derived from Gauss's Variational Equations (GVEs) [8]. Unlike approaches that rely on mean orbital elements over fixed intervals, this method computes the instantaneous drift of both secular and osculating orbital elements. The manoeuvre model is directly coupled with the high-fidelity lunar numerical propagator, ensuring that the perturbed state of LNSS satellites reflects the full spectrum of dynamic

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perturbations responsible for orbital element drift. By embedding these effects within the propagation framework, the approach yields more accurate and reliable ΔV estimates than those obtained from simplified analytical techniques.

This study employs two distinct classes of optimization techniques, evolutionary algorithms and pattern-based Pareto search. Evolutionary methods, represented by Non-dominated Sorting Genetic Algorithm II (NSGA-II), rely on stochastic operators such as crossover and mutation to explore a wide and potentially discontinuous design space. NSGA-II improves upon classical Genetic Algorithm (GA) by incorporating a non-dominated sorting scheme and crowding distance assignment, which enhances convergence towards the Pareto front while maintaining population diversity [9]. In contrast, the pattern-based Pareto search method is deterministic, systematically sampling the design space on a progressively refined mesh to identify Pareto-optimal solutions. The evolutionary algorithms emphasize global search capability and robustness against local minima, whereas pattern search methods prioritize precision and reproducibility but face increasing computational cost as the problem dimensionality expands [10]. By comparing these approaches, this work also aims to evaluate their relative strengths in addressing the wide orbital search space and multi-objective trade-offs associated with LNSS constellation design.

The primary aim of this study can be summarized as:

I. Broaden the orbital design space to include both frozen and non-frozen orbits.
II. Formulate the GVE-based manoeuvre model for reliable ΔV computation.
III. Conduct a comparative analysis of an evolutionary algorithm versus a pattern-based design method to assess their effectiveness in balancing computational efficiency with solution quality.

The paper is organized as follows: Section 2 explains framework computation of Figure of Merits (FOMs). Section 3 details the optimization logic that describes the simulation setup, highlighting the integration of the optimization framework with the FOMs and orbital dynamic model. Section 4 presents the simulation results, first through a comparative analysis of key competing FOMs as proposed by NSGA-II and Pareto search algorithms. It is then followed by comparative analysis of the design space of both algorithms. Finally, Section 6 summarizes the conclusions and outlines future research directions.

2. Material and methods

The objective function of this optimization problem is categorized into design (search) space and FOM. These are explained in a detail as follows:

2.1. Search Space

The design space is formulated directly in terms of the orbital elements, which represent the geometry of potential LNSS constellations. The optimization process explores variations in parameters such as semi-major axis (a), eccentricity (e), inclination (i), right ascension of the ascending node (Ω), argument of periapsis (ω), number of orbital planes (N_p) and number of satellites per plane (N_s) thereby encompassing both frozen and non-frozen orbital configurations. The N_p and N_s are derived by mean anomaly (M) and Ω uniform separation in a walker configuration [11]. Defining the problem in the orbital-element domain allows the optimization algorithm to efficiently map design trade-offs, as these parameters directly influence the FOMs. We have defined the limits of the search space to ensures that the optimization results are physically interpretable while enabling a systematic examination of feasible lunar orbits. For instance, the upper bound of a is 24000 km as the orbits beyond this limit becomes distant lunar orbits. The orbital elements range is represented by upper and lower bounds is depicted in Table 1.

2.2. Figure of Merit

PDOP and HDOP: The positioning performance of the proposed LNSS is assessed using DOP metrics, which capture the impact of satellite-receiver geometry on positional accuracy. Specifically, the analysis focuses PDOP for three-dimensional positioning and HDOP for horizontal accuracy. To systematically evaluate DOP across the lunar surface, the south polar region is discretized into a uniform latitude-longitude grid spanning latitudes from 60°S to 90°S and longitudes from 0° to 360°, with a grid resolution of 10° in both latitude and longitude directions, as shown in Fig.1. At each discrete surface point and time epoch, the visibility of each satellite is assessed against a minimum elevation angle constraint (5°), and only grid points with at least 4 visible satellites are used to construct the geometric configuration matrix required for DOP evaluation [12]. The resulting DOP values are collected over the full spatiotemporal domain and filtered using a 3-sigma statistical method to remove outliers, producing mean PDOP ($PDOP_{3\sigma\text{-}mean}$) and mean HDOP ($HDOP_{3\sigma\text{-}mean}$) values that represent typical positioning performance.

PDOP and HDOP availability: In addition, an availability metric is computed by calculating the percentage coverage of grid points and time steps where the DOP remains below a defined threshold,

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providing a quantitative measure of the system's ability to deliver reliable positioning throughout the lunar south polar region. This methodology ensures that both the spatial variability due to satellite-receiver geometry and the temporal variations induced by orbital motion and lunar rotation are captured, offering a comprehensive assessment of LNSS performance and navigation service reliability.

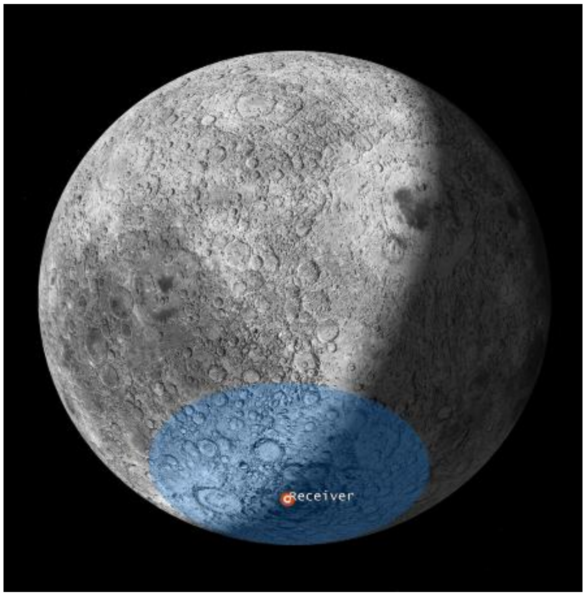

Fig. 1. Area of Interest (AOI) for navigation performance analysis

Station-keeping ΔV budget: It is the fundamental consideration in LNSS constellation design, as it directly influences propellant requirements and the operational lifetime of the system. In this study, ΔV estimation is closely integrated with the lunar propagator. The propagated position and velocity of the satellite is converted into classical orbital elements to quantify natural drift caused by perturbations, including lunar gravitational harmonics, third-body effects from Earth, Sun and Jupiter and solar radiation pressure. Secular variations in semi-major axis, eccentricity, inclination, RAAN, and argument of periapsis are then analysed and mapped into thrust requirements using a semi-analytical approach based on Gauss's Variational Equations (GVEs). The mathematical model of GVEs can be found in reference [8].

The thrust is resolved in the Radial-Tangential-Normal (RTN) frame. Tangential thrust primarily adjusts orbital energy and semi-major axis, radial thrust addresses eccentricity and argument of periapsis, and normal thrust modifies the orbital plane orientation, controlling inclination and RAAN. For simplification and model stability, semi-major axis control is assumed to be dominated by tangential acceleration, while eccentricity and argument-of-periapsis corrections are combined into a single radial component. The instantaneous thrust magnitude defines the immediate effort required to maintain the orbit against perturbations. By integrating this over the full propagation period, the total station-keeping ΔV budget is obtained, representing the cumulative fuel cost to maintain each satellite in its nominal configuration.

The FOMs are also bounded within desired ranges to only search the feasible solution offering acceptable navigation performance and ΔV per satellite per year. The purpose of this constraint is to ensure efficiency of the optimization framework. The values of these bounds are depicted in Table 1.

Table 1: Upper and lower bounds of design space variables and FOMs

		Type	**Lower bound**	**Upper bound**
Design space Variables	a	Real	4000 km	24000 km
	e	Real	0	0.8
	Ω	Real	0°	359°
	ω	Real	1°	359°
	i	Real	1°	90°
	N_p	Integer	1	4
	N_s	Integer	1	4
FOM	Total satellites	Integer	1	16
	$PDOP_{3\sigma\text{-}mean}$	Real	> 0	10
	$PDOP_{\text{availability}}$	Real	70	100
	$\Delta V_{\text{station-keeping}}$	Real	> 0 kms^{-1} per year	1 kms^{-1} per year

3. Optimization framework

The first step involves propagating the LNSS constellation configurations in Moon Centred Inertial (MCI) frame within the specified orbital bounds, as summarized in Table 1, using the dynamic propagation model. The propagator setup parameters are depicted in Table 2. In the second step, the DOP metrics are computed in Moon Centred Moon Fixed frame (MCMF) across the lunar polar grid. Similarly, the orbital state of the first of each of the LNSS configuration is extracted and further processed to calculate station-keeping ΔV budget. The final stage consolidates all of performance metrics into a compact fitness vector with well-defined constraints. A multi-objective optimization algorithm then identifies Pareto-optimal solutions, revealing trade-offs between positioning accuracy, spatial coverage, fuel expenditure, and satellite count, thereby guiding the selection of efficient and operationally feasible LNSS constellation designs. The schematic diagram of the optimizer in depicted in Fig.2.

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Table 2: Lunar Propagator setup parameters

Propagator	High Precision Lunar Orbit Propagator (LHPOP) [13]
Lunar gravity model	Moon_AIUB-GRL350B [14] Degree and order: 8
Perturbations	Third body (Sun, Earth and Jupiter), SRP, relativistic corrections and Earth albedo
Frames of reference	Inertial MCI and MCMF
Integrator	Runge–Kutta (4,5)
Propagation Time	3.86 days with 900-s step size

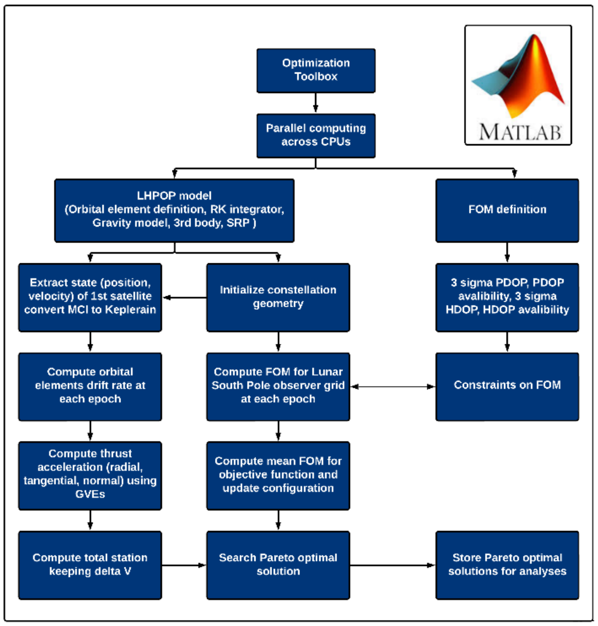

Fig. 2. Schematic diagram of the optimization framework in MATLAB

3.1. NSGA-II

The optimization is carried out using a genetic algorithm based on the NSGA-II framework, which is particularly suited for multi-objective problems with competing performance metrics. The process begins with the generation of an initial population of 200 candidate constellations, each defined by normalized design variables representing both orbital elements and constellation architecture. For every candidate, orbital propagation and FOM computation are performed to evaluate navigation performance alongside the associated station-keeping cost. The population has been evolved over 500 generations using evolutionary operators. Crossover, applied with a fraction of 0.75, combines attributes of high-performing solutions to explore promising regions of the design space. Mutation, guided by an adaptive feasible scheme, introduces controlled variability to maintain diversity and avoid premature convergence. A Pareto fraction of 0.35 is imposed to retain a balanced set of nondominated solutions. At each generation, solutions are ranked through nondominated sorting, and elitism ensures the preservation of the best-performing candidates. This process progressively refines the search, yielding a well-distributed Pareto front that captures the trade-offs between constellation size, navigation accuracy, and fuel expenditure.

3.2 Pattern search based approach

The second optimization strategy applied in this study is a Pareto search algorithm, which is a pattern search derivative-free approach for exploring complex multi-objective landscapes. Unlike the GA, which relies on stochastic variation to evolve candidate constellations across generations, Pareto search systematically explores the design space through adaptive mesh refinement within defined tolerance level. This makes it particularly effective for problems where objective functions are non-linear. For this study, the search was configured with a maximum Pareto set size of 5000 solutions and 100,000 function evaluations ensure convergence toward high-quality non-dominated solutions.

The key distinction of both algorithms lies in the search methodology. The GA emphasizes global exploration by maintaining a diverse pool of solutions that can leap across widely separated regions of the design space. On the other hand, Pareto search emphasizes local refinement, adaptively concentrating its effort on promising regions to sharpen the quality of solutions. Collectively, the two methods provide complementary strengths, e.g. GA is well suited to discovering a broad range of candidate architectures without being trapped in local optima, whereas Pareto search is more efficient at precisely delineating trade-offs once those regions are identified. The comparative analysis of both approaches provides a clear understanding of the trade-offs while highlighting the strengths and limitations of each algorithm.

The other distinction between the two approaches lies in the degree of control over operation. For example, in the genetic algorithm, the optimizer allows explicit tuning of evolutionary operators, (crossover fraction, mutation strategy, and population size) thereby offering fine-grained control over exploration and exploitation of the design space. This flexibility enables the algorithm to be tailored to the specific structure of the optimization problem and to balance convergence speed with solution diversity. By contrast, the pattern search method provides a more constrained environment, where the user can primarily adjust high-level parameters such as the maximum size of the Pareto set and the total number of function evaluations. It limits the ability to directly influence the search dynamics. As a result, the genetic algorithm offers

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greater adaptability for the problems whereas Pareto search emphasizes robustness and ease of use.

4. Results

4.1. *Trade-off analysis*

The NSGA-II optimization process produced 4849 unique LNSS constellation configurations out of 100,000 processed data sets that satisfying FOM constraints. Fig.3 depicts the Pareto front across key competing FOMs, including PDOP, HDOP, the annual station-keeping ΔV budget, and the overall mission cost measured by the number of satellites. From mission design perspective, the most competitive designs are the solutions represented by of less than 0.2 kms^{-1} per year situated along the densely populated lower boundaries of Pareto front. Within this region, mission designers can identify a spectrum of trade-offs between navigation precision and constellation size, offering flexibility to balance fuel efficiency, coverage quality, and system cost depending on mission priorities.

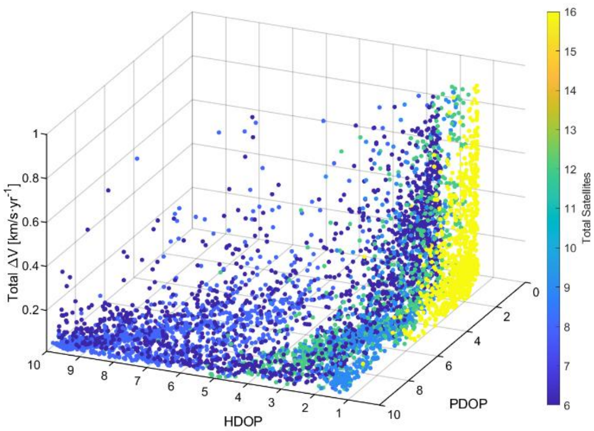

Fig. 3. Pareto front between HDOP, PDOP and station-keeping ΔV (Total satellites as color-coded) for NSGA-II algorithm.

The Pareto search algorithm identified 3,468 feasible solutions, with the resulting front for PDOP, HDOP, and the annual ΔV budget presented in Fig 4. A clear distinction emerges when comparing the fronts obtained from the two optimization strategies. The GA-based solutions span the full range of the FOM bounds, thereby capturing a broad diversity of constellation designs. By contrast, the nature of the Pareto search algorithm, drives the exploration toward locally efficient regions of the design space. This systematic refinement concentrates the solutions at lower ΔV and HDOP values, producing a narrow yet highly efficient front but at the cost of reduced diversity. The comparison highlights that while both algorithms are effective in generating pareto fronts, the GA offers a broader distribution of alternatives and faster convergence, making it more advantageous for exploratory constellation design studies.

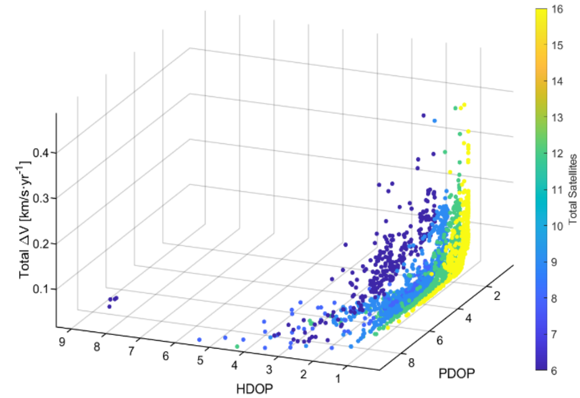

Fig. 4. Pareto front between HDOP, PDOP and station-keeping ΔV (Total satellites as color-coded) for Pareto search (pattern search) algorithm.

4.2 Orbital configuration

The LNSS design spaces obtained from both optimizers are shown in Fig. 5 and Fig. 6. Distinct clustering patterns emerge between the two approaches. The NSGA-II solutions span a wide range of orbital elements particularly semi-major axis, inclination, and argument of perigee while eccentricity remains concentrated between 0.6 and 0.8. This broader coverage enables the identification of diverse and efficient configurations, including those that maximize satellite visibility over the lunar south pole.

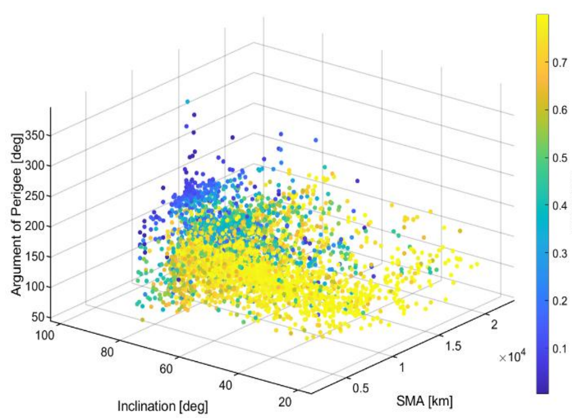

Fig. 5. Orbital elements representation of the LNSS solutions in 4-objective space for NSGA-II algorithm.

In contrast, the pattern-based Pareto search solutions are confined to specific orbital regions selected by the algorithm. While these clusters correspond to favourable FOM values, the search lacks diversity, limiting the range of alternative constellation geometries. Moreover, the orbital elements are confined within a lower search space. Comparing Fig. 5 and Fig. 6, the semi-major axis of the solutions lies within 10,260 km – 24,000 km compared to 4,530 km

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- 23,980 km for NSGA-II. The similar difference can also be found for inclination and argument of perigee data of the two algorithms. Despite the limited design space, the Pareto search method has an advantage of lower ΔV (less than 0.43 kms^{-1} / year).

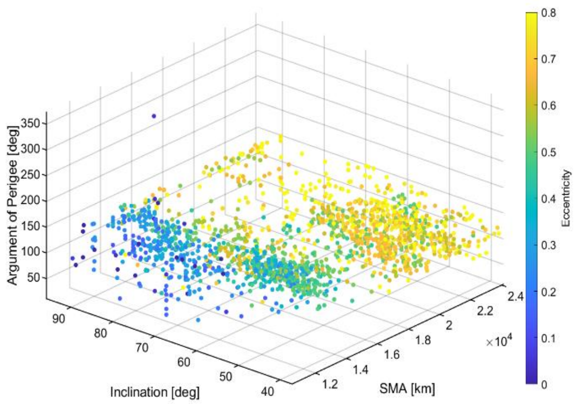

Fig. 6. Orbital elements representation of the LNSS solutions in 4-objective space for Pareto Search algorithm.

4.3 Sample Configurations

Table 3 presents a selection of Pareto optimal solutions identified by both algorithms, highlighting the corresponding orbital geometries and FOM values under the constraints of PDOP and HDOP availability above 90% and a per-satellite station-keeping budget of $\Delta V \leq 1$ kms^{-1} per year. While all listed configurations represent balanced trade-offs, Solution 8 stands out for providing high positioning performance with fewer than ten satellites. Similarly, Solutions 1 and 6 achieve the lowest constellation sizes. It is important to note that many other viable constellations satisfy the mission design objectives; the cases shown here serve as representative examples to illustrate the relationships among navigation accuracy, constellation size and orbital maintenance.

As there are many solutions in table 3, we have selected one of the solutions for the demonstration of orbital geometric arrangement and FOM computation. The 3-D model of solution 8 in MCI frame is presented in Fig. 7. The FOM values computation process of this sample configuration has also been depicted in Fig. 8, 9 and 10. The temporal variation of the mean satellite visibility, PDOP and HDOP of the spatiotemporal observer grid between 60°S - 90°S are depicted in the figure Fig. 8. The PDOP and HDOP excellent are stable for the entire simulation span due to uniform high satellite visibility. Similarly, the progression of station-keeping ΔV with orbital evolution of the first satellite of the constellation and thrust acceleration have been presented in Fig. 9 and 10, respectively.

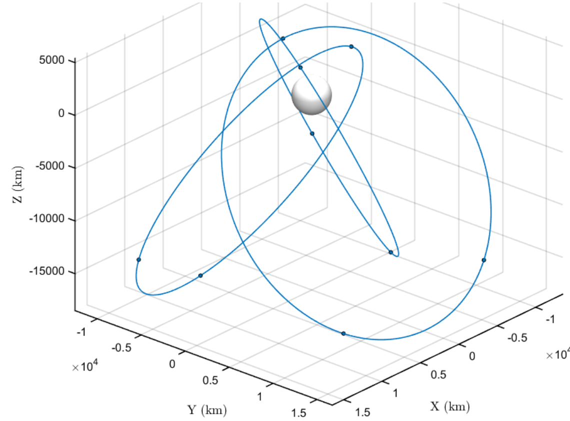

Fig. 7. Orbital configuration visualization of solution 8 in MCI frame.

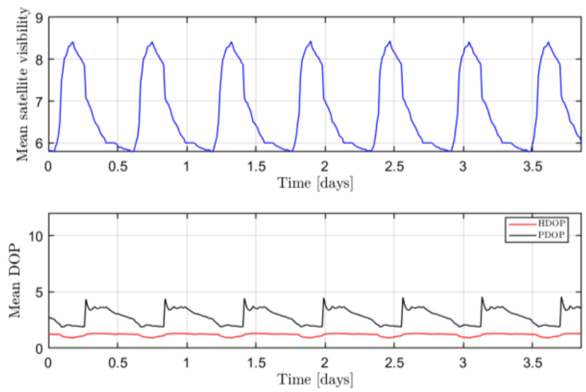

Fig. 8. Temporal variation in mean visibility, PDOP and HDOP for the observer grid on south pole as calculated for solution 8.

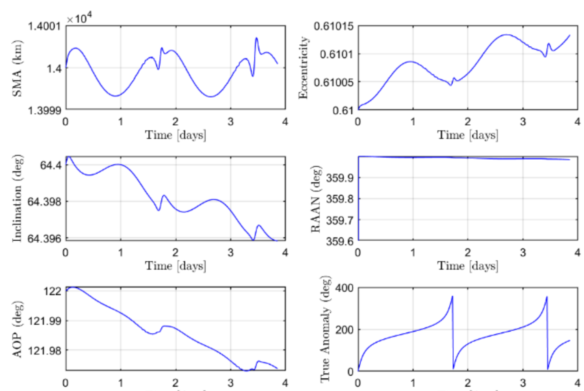

Fig. 9. Evolution of orbital elements of the first satellite of solution 8 under the influence of perturbations in the MCI-J2000 frame.

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Table 3: The design and mean FOM values of 10 selective LNSS configurations searched by the NSGA-II and Pareto search algorithms.

	ID	*a (km)*	*e*	*ω (deg)*	*I deg*	N_p	N_S	**No of Sats**	**PDOP$_{3\sigma}$**	***PDOP availability %***	**HDOP$_{3\sigma}$**	***HDOP availability %***	*ΔV kms^{-1}/ year (Total)*
NSGA-II	1	16106	0.62	65	85.8	2	3	6	9.4	99	8.6	99	0.06
	2	13195	0.74	106	86.0	2	4	8	7.2	100	6.5	100	0.14
	3	16301	0.76	100	85.3	2	3	9	4.5	98	1.7	100	0.15
	4	21114	0.77	111	27	4	3	12	5.8	98	0.8	100	0.12
	5	10529	0.72	93	62.7	4	4	16	3.0	100	0.8	100	0.14
Pareto Search	6	21658	0.66	62	85.3	2	3	6	8.7	99	8.1	99	0.07
	7	16905	0.70	101	51.9	2	4	8	4.3	100	1.1	100	0.08
	8	**14000**	**0.61**	**122**	**64.4**	**3**	**3**	**9**	**2.6**	**100**	**1.1**	**100**	**0.07**
	9	18843	0.47	103	54.0	3	4	12	2.2	100	0.94	100	0.05
	10	13716	0.45	70	70.0	4	4	16	1.7	100	0.96	100	0.04

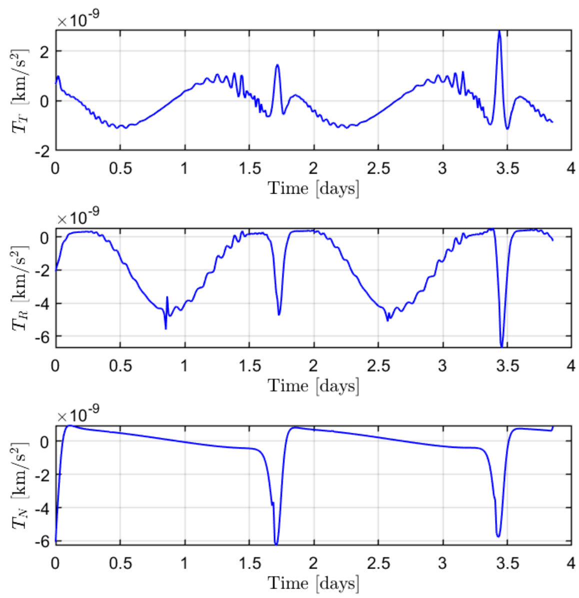

Fig. 10. Instantaneous thrust magnitude required for station-keeping for the first satellite of solution 8 LNSS constellation.

6. Conclusions

In this study, a comprehensive multi-objective optimization framework has been developed and validated for the design of a dedicated LNSS. The framework couples a high-fidelity lunar propagator, including realistic perturbations, with a semi-analytical continuous thrust ΔV model. This formulation provides more precise estimates of orbital drift and correction costs compared to earlier simplified approaches.

Navigation performance was assessed through PDOP and HDOP analyses over a south pole observer grid. Optimization is performed using two different approaches including NSGA-II and pattern-based Pareto-search. Both algorithms explored a six-dimensional design space and revealed Pareto fronts capturing trade-offs between key FOMs like constellation size, positioning accuracy and station-keeping ΔV budget. The results of both optimizers identified numerous compact non-frozen elliptical constellations capable of achieving PDOP < 10 with ≥70% availability and $\Delta V \leq 1$ km/s/year. The findings highlight that orbital geometries optimized for navigation accuracy may increase ΔV demands and vice versa, underscoring the importance of balanced trade-offs in constellation design. The comparative analysis of optimizers shows that GA algorithm ensures diversity in the solutions with lower computation time. Conversely, Pareto search method is computationally expensive and lacks diversity but finds solutions / regions that represent highest possible quality FOMs.

Although the current work focuses on the lunar south pole, the results demonstrate that fewer satellites may be sufficient to achieve mission goals relative to prior studies. Looking ahead, expanding the framework to incorporate onboard orbit determination, time-transfer algorithms, robustness to injection errors, heterogeneous propulsion systems, and autonomous maneuver planning will further enhance its applicability. Moreover, incorporating integration studies with Earth-based GNSS constellations offers a promising avenue for the realization of resilient and cost-effective lunar navigation infrastructures.

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