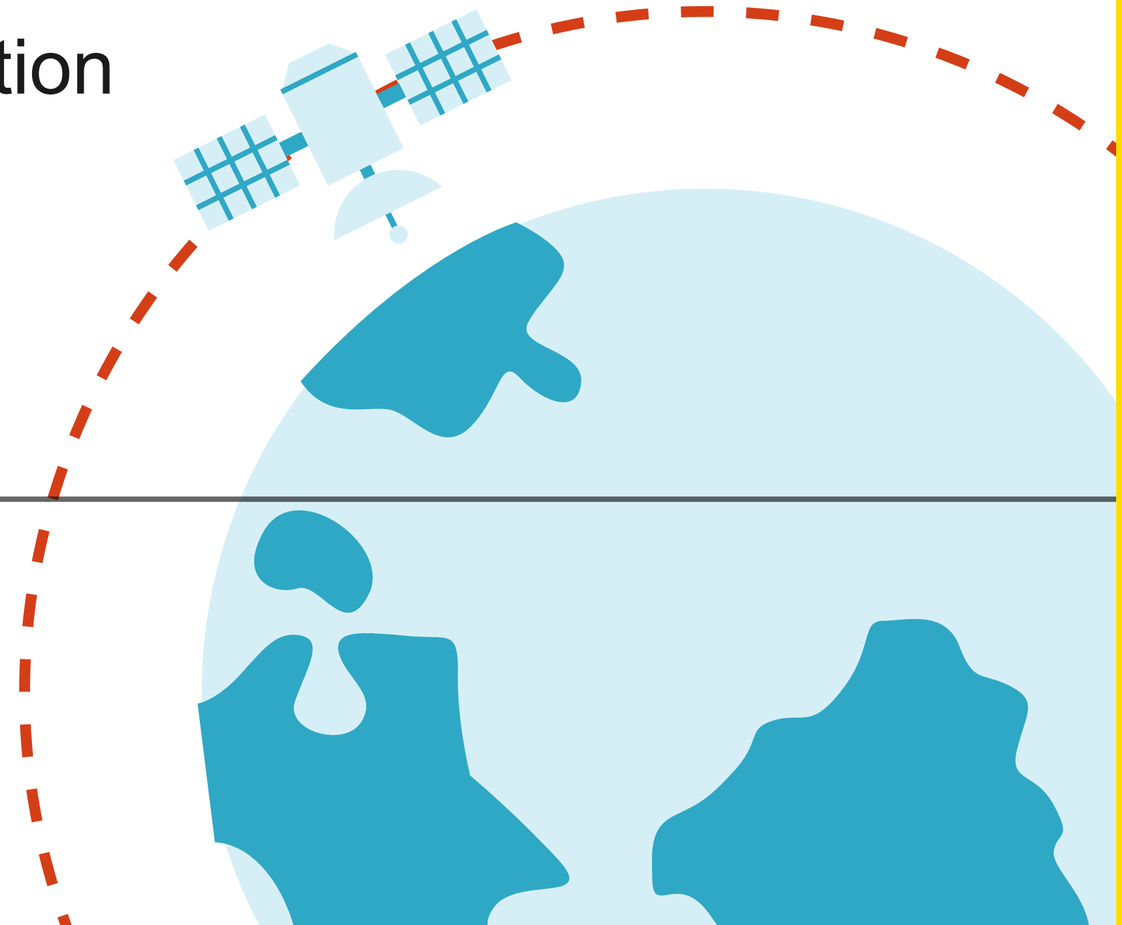
A Range/Range Constrained Admissible Region Approach in GEO

Tracklet-to-Orbit Association



Honours Thesis

Supervised by Dr Yang Yang
Written by Angela Ng

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Overview

01 The Crowded Space Environment

02 Problem Definition - Too-Short-Arc & Manoeuvred Objects

03 Literature Review

04 Theoretical Framework

05 Methodology - Lambert range/range Solution

06 Results - Simulated and Real

07 Ongoing Work

08 Conclusion

Introduction

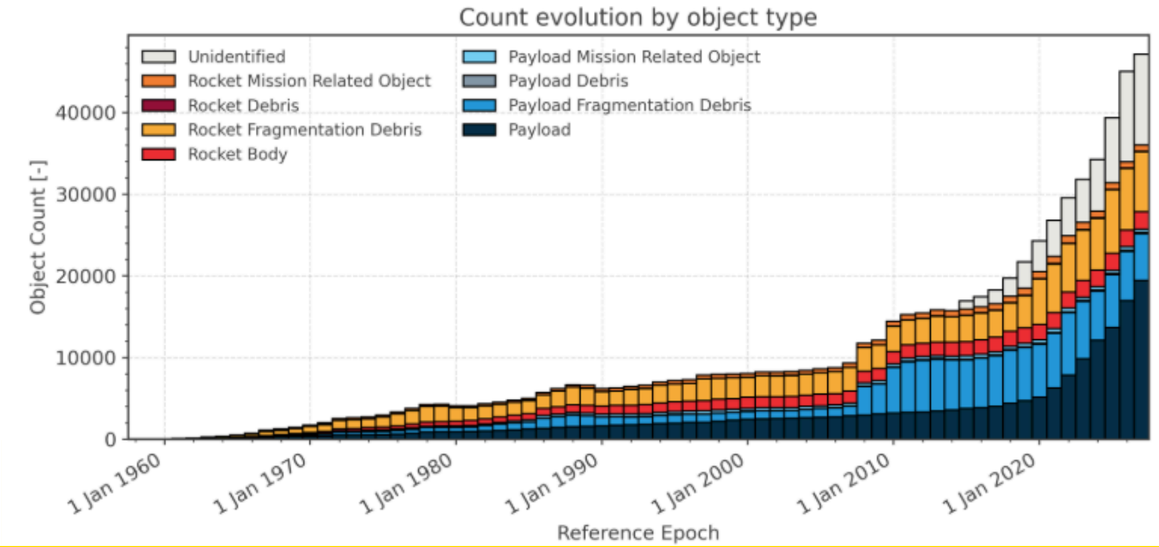
Crowded Space Environment

European Space Agency Space Debris Environment, 2026 reported [1] :

- 47,140 number of tracked objects
- More than 660 break-ups, explosion, collision or anomalous events
- 1.5 million space debris in orbit

Objects in Orbit

Image by ESA Space Debris Environment [1]



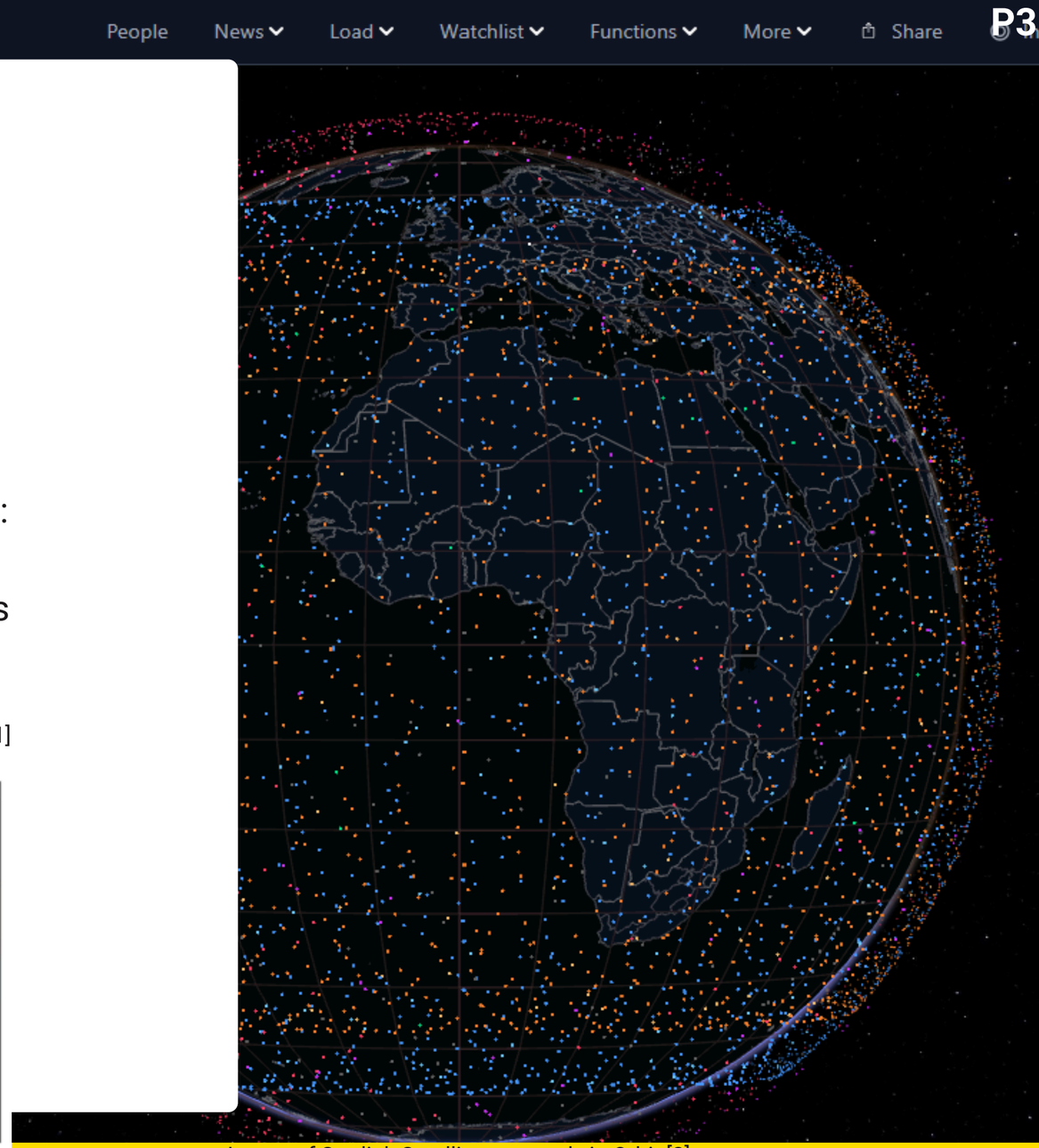


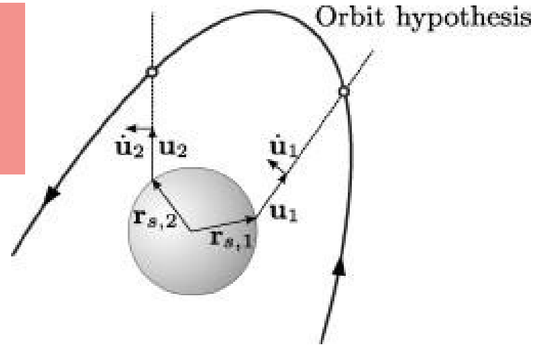
Image of Starlink Satellites currently in Orbit [2]

[1] ESA Space Debris Office, 'ESA'S ANNUAL SPACE ENVIRONMENT REPORT', 2025. Accessed: Apr. 01, 2026. [Online]. Available: https://www.sdo.esoc.esa.int/environment_report/Space_Environment_Report_latest.pdf
[2] "Satellite Tracker — Live Map of Starlink & 30,000+ Satellites," satellitemap.space. https://satellitemap.space/ (accessed Nov. 3, 2026)

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Problem Definition

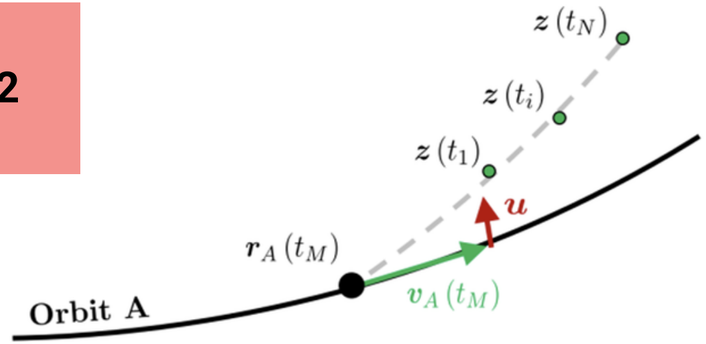
01



Tracklet association problem, requiring two tracklets for two LOS [3]

Too-Short-Arc Problem

02



Tracklet-to-Orbit Manoeuvre Estimation illustration by [4]

Manoeuvred Objects

01

Manoeuvre Detection

Did a catalogued object manoeuvre?
If so, is the tracklet consistent to it?

02

Tracklet-to-Orbit Association

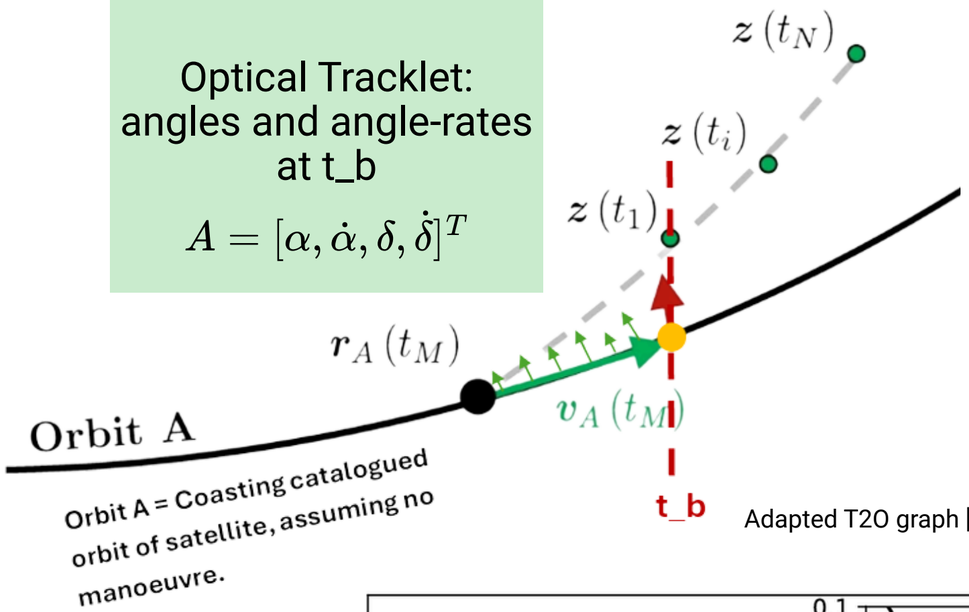
Which Catalogued Object, if any, produced the tracklet?

[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.
[4] A. Pastor, G. Escribano, M. Sanjurjo-Rivo, and D. Escobar, 'Satellite maneuver detection and estimation with optical survey observations', J. Astronaut. Sci., vol. 69, no. 3, pp. 879–917, Jun. 2022, doi: 10.1007/s40295-022-00311-5.

Missing 2 of the 6 state dimension unobserved (ρ, ρ̇)

Optical Tracklet: angles and angle-rates at t_b

$$A = [\alpha, \dot{\alpha}, \delta, \dot{\delta}]^T$$



Adapted T2O graph [4]

Range/Range-rate CAR Continuous Energy-Optimal Control Theory

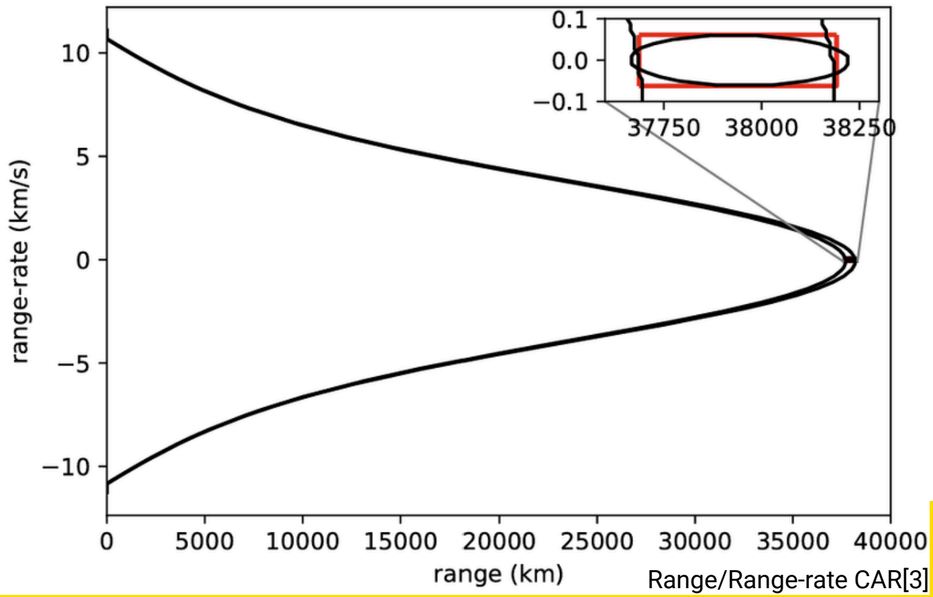
Romain Serra, Carlos Yanez, Carolin Frueh

Date: April 2021

Too-Short-Arc Problem:

- Range/range-rate pairs.
- Initial Value Problem (IVP)

IVP inferred: (ρ, ρ̇) giving complete 6-D state at one epoch both for t_b.



Range/Range-rate CAR[3]

Semi-major axis (km)	42,165.267
Eccentricity	0.0004854
Inclination (deg)	0.0666
Argument of perigee (deg)	97.923
Right Ascension of ascending node (deg)	28.890
Longitude (deg)	18.750 East

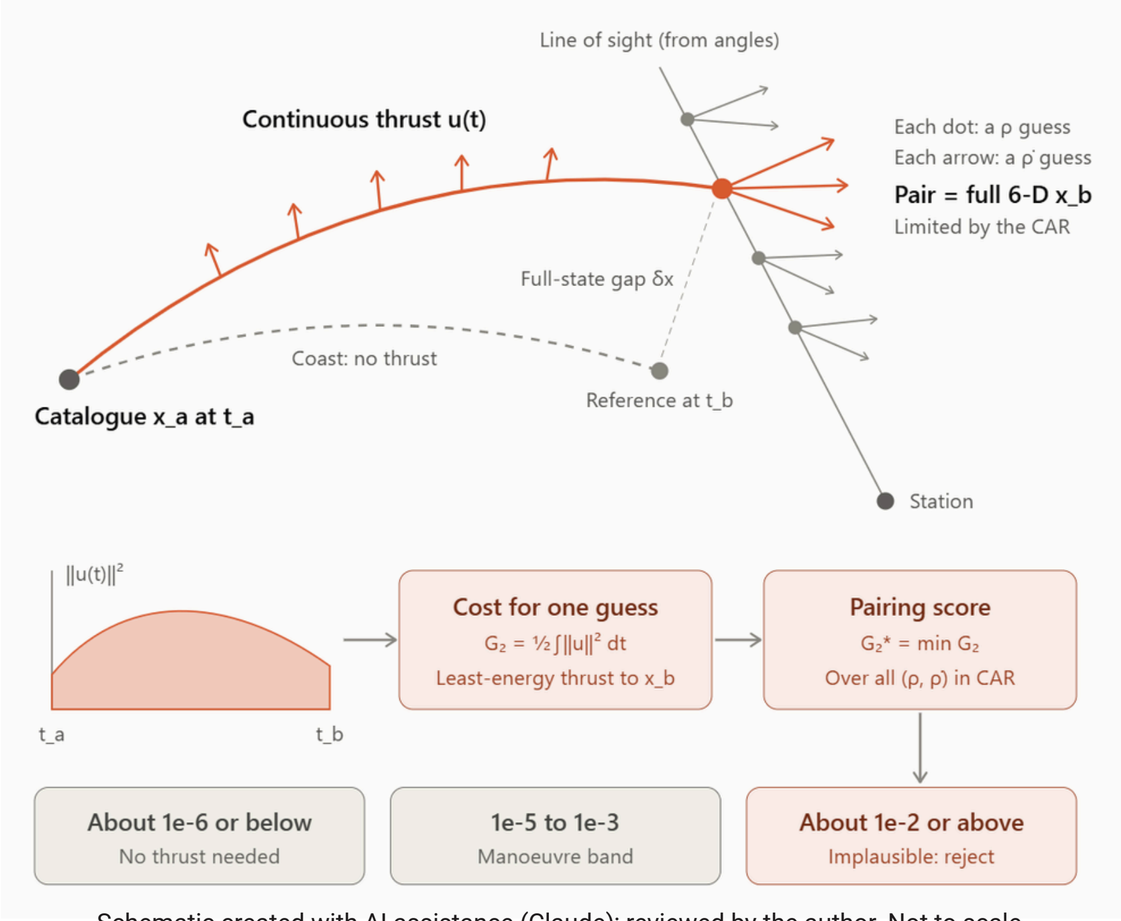
Nominal Orbit [3]

[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.
[4] A. Pastor, G. Escribano, M. Sanjurjo-Rivo, and D. Escobar, 'Satellite maneuver detection and estimation with optical survey observations', J. Astronaut. Sci., vol. 69, no. 3, pp. 879–917, Jun. 2022, doi: 10.1007/s40295-022-00311-5.

Range/Range-rate CAR Continuous Energy-Optimal Control Theory

Romain Serra, Carlos Yanez, Carolin Frueh

Date: April 2021



Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.

Manoeuvred Objects:

How much effort would this link require?

An alternative to the fuel expenditure [3]:

$$G_2(\mathbf{u}) = \frac{1}{2}\int_{t_a}^{t_b} \|\mathbf{u}(\tau)\|_2^2 \, d\tau$$

is the energy cost.

Least control effort that connects the catalogue state to the tracklet.

[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.

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Limitations & Research Extensions

High Noise Floor
Measurement noise significantly influences the results.
Causing wide spread of results.

ΔV-Based Metric
G_2 as a relative ranking score, not a physical manoeuvre-energy or impulsive- ΔV estimate.

Comparison to Boundary Value Problem (BVP)
Both works sample for range/range-rate pairs.

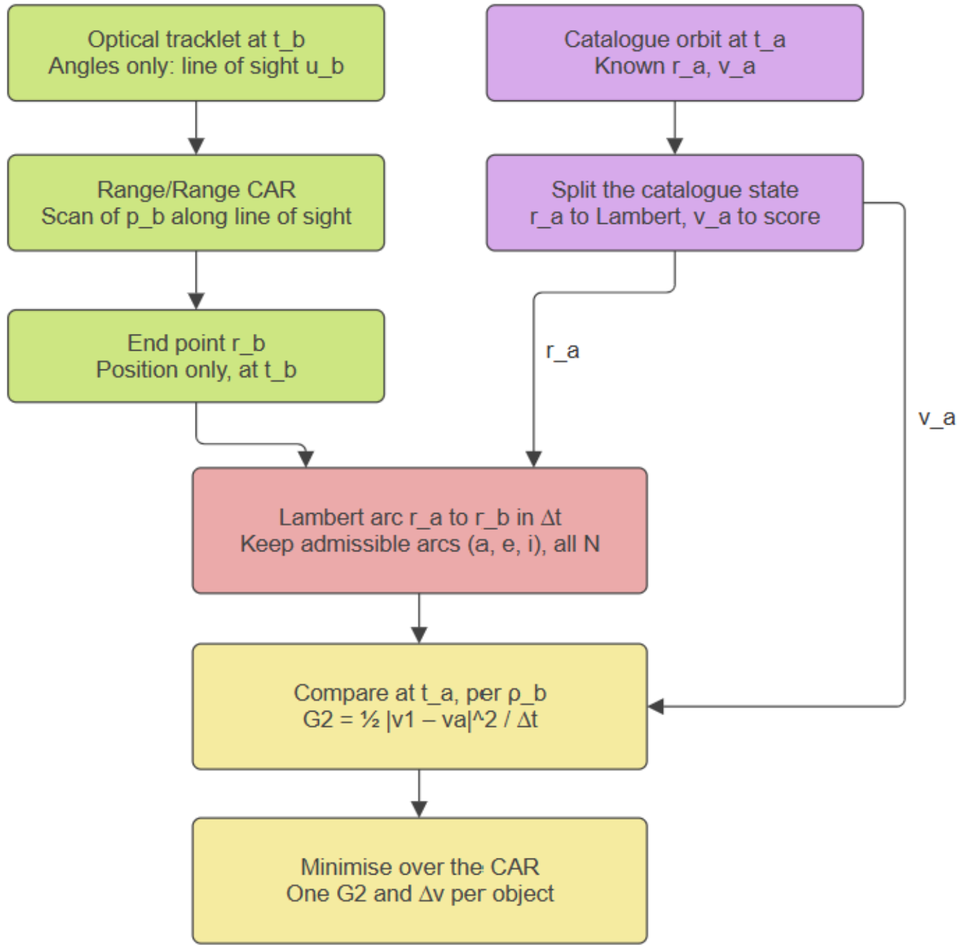
Hypothesis and Predictions

	Boundary Condition	Control Model	Initial Guess
Serra et al.	Range/Range-rate	Continuous Burn	Needed
This Methodology	Range/Range	Impulsive Burn	Not Needed

	Prediction	Test	Supported if
H1	The BVP metric is more concentrated than IVP metric.	Compare the M1 spread for both methods using the same station, noise, and cases.	The BVP spread is smaller across epochs by at least 5 magnitudes.
H2	The BVP separates M1 from M2-M5, but IVP does not.	Apply one threshold set from the M1 distribution and compare detection and false-alarm rates.	The BVP meets is able to conclusively separate M2-M5 manoeuvred.

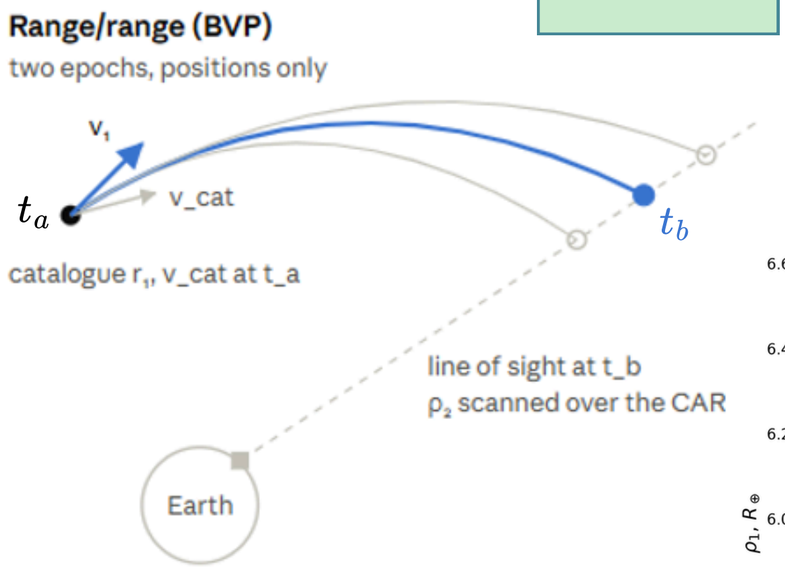
Lambert Range/Range BVP

Methodology

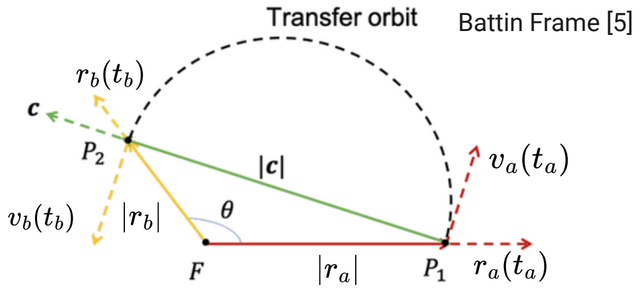


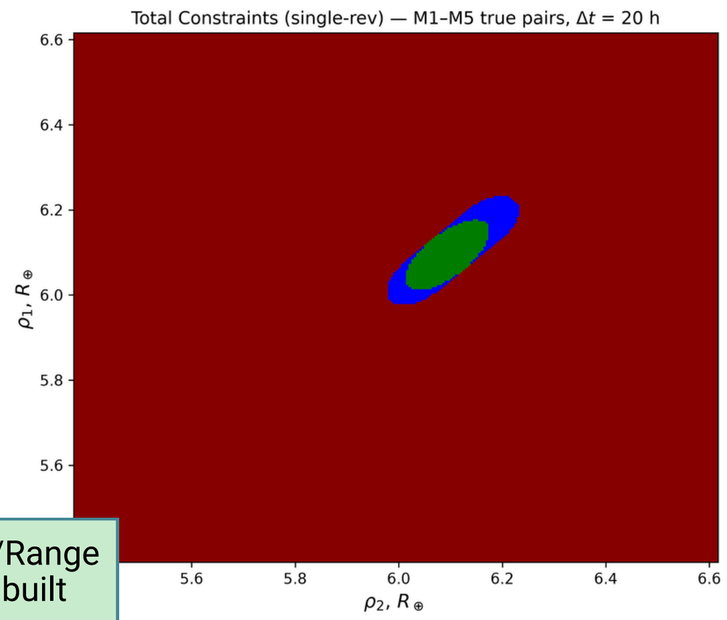
Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.

Lambert Arc



Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.





Range/Range CAR built

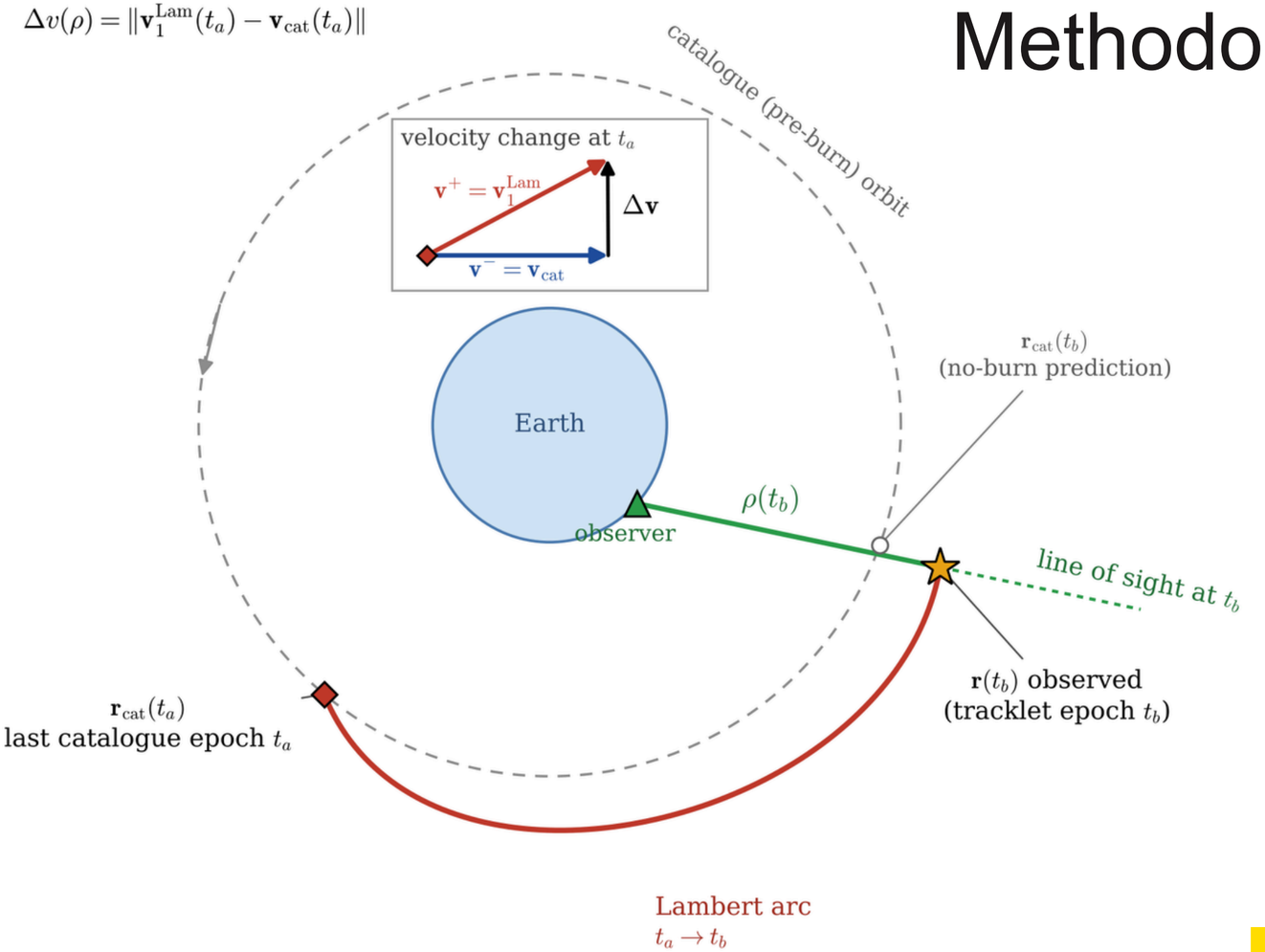
[5] Y. Yang, Z. Li, H. Cai, and Y. Zhang, "Single-/multiple-revolution Lambert's problem constraints for optical track-to-track association," AAS preprint 21-671.

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Impulsive Burn at t_a

Methodology

$$\Delta v(\rho) = \|\mathbf{v}_1^{\mathrm{Lam}}(t_a) - \mathbf{v}_{\mathrm{cat}}(t_a)\|$$



$$\Delta v_{t_a}(\rho) = \left\|\mathbf{v}_1^{\mathrm{Lambert}} - \mathbf{v}_{\mathrm{cat}}(t_a)\right\|$$

$$G_2 = \frac{1}{2}\frac{|\Delta v_{t_a}(\rho)|^2}{TOF}$$

$$TOF = t_b - t_a$$

Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.

Simulated Data Sets

Error Case	# Obs	Time Step (s)	Obs. Noise (arcsec)	Tracklet Noise (arcsec)
T1	3	20	0.7	0.404
T2	11	4	0.27	0.081

Simulated Noise [3]

M1

No Manoeuvre

$\Delta V_{true} = 0 m/s$

M2

North-South Impulsive Burn

$\Delta V_{true} = 2 m/s$

North-South Station Keeping

M3

Tangential Impulsive Manoeuvre

$\Delta V_{true} = 1.448 m/s$

Longitude shift for eccentric drift correction.

M4

Continuous Burn

Longitude shift with circular drift (0.51 deg/day) correction.

$Total\ \Delta V_{true} = 1.448 m/s$

Continuous manoeuvre for 72 minutes

M5

Continuous Burn

Longitude shift with circular drift (1.28 deg/day) correction.

$Total\ \Delta V_{true} = 3.634 m/s$

Across two continuous manoeuvres, each undertaken for 3 hours.

Simulated Data sets [3]

[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.

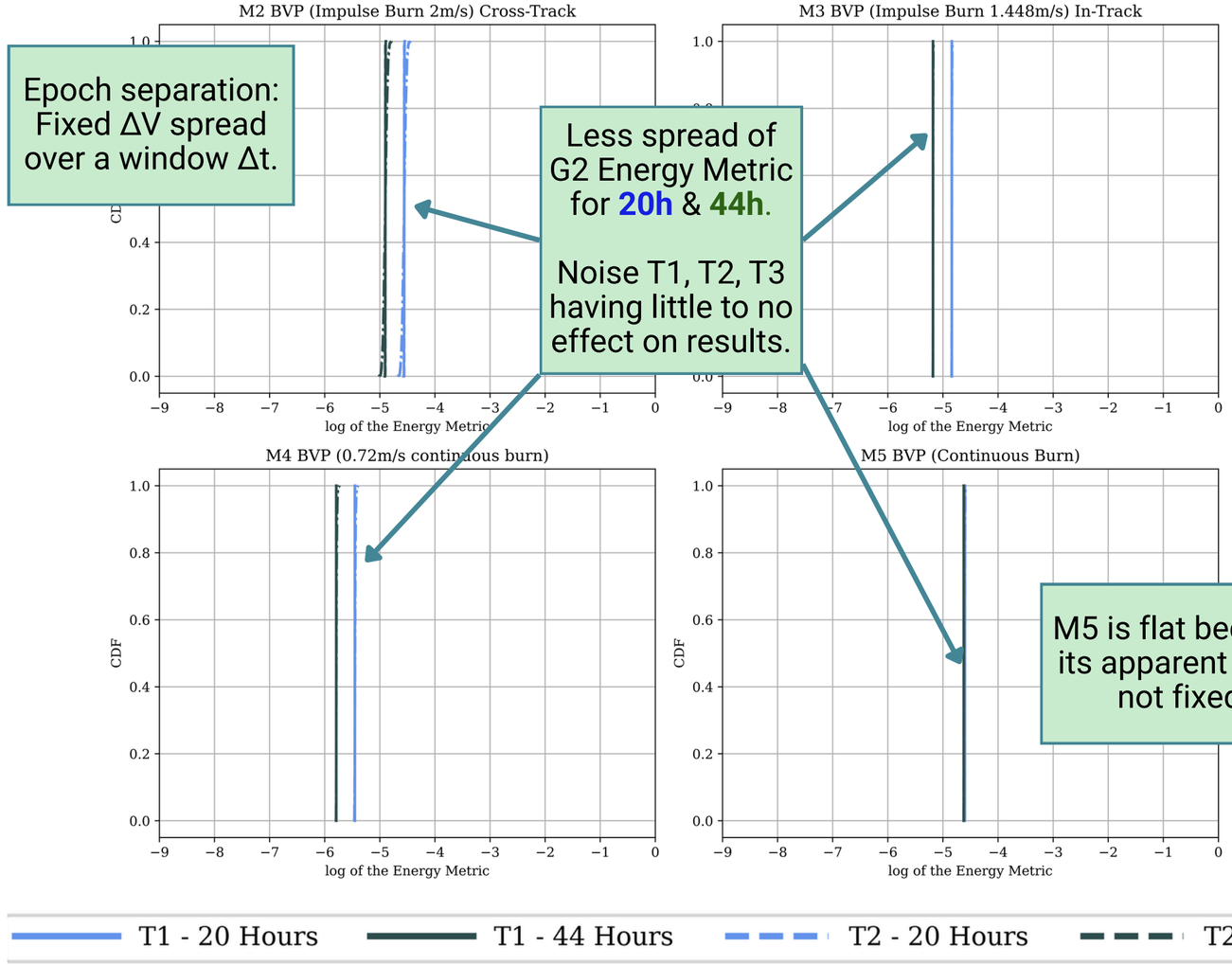
BVP vs. IVP

Properties	BVP	IVP
Spread of Energy Metric	Concentrated Energy Metric, noise has limited affect	Large Spread across several magnitudes
Detecting Maneouvre	Successful separation between M1 no-maneouvre spread and M2-M5 maneouvred cases	Unable to separate between M1-M5

Simulated Results M2-M5 t_a

Separated by only time epoch **20h** & **44h**

Our BVP Results



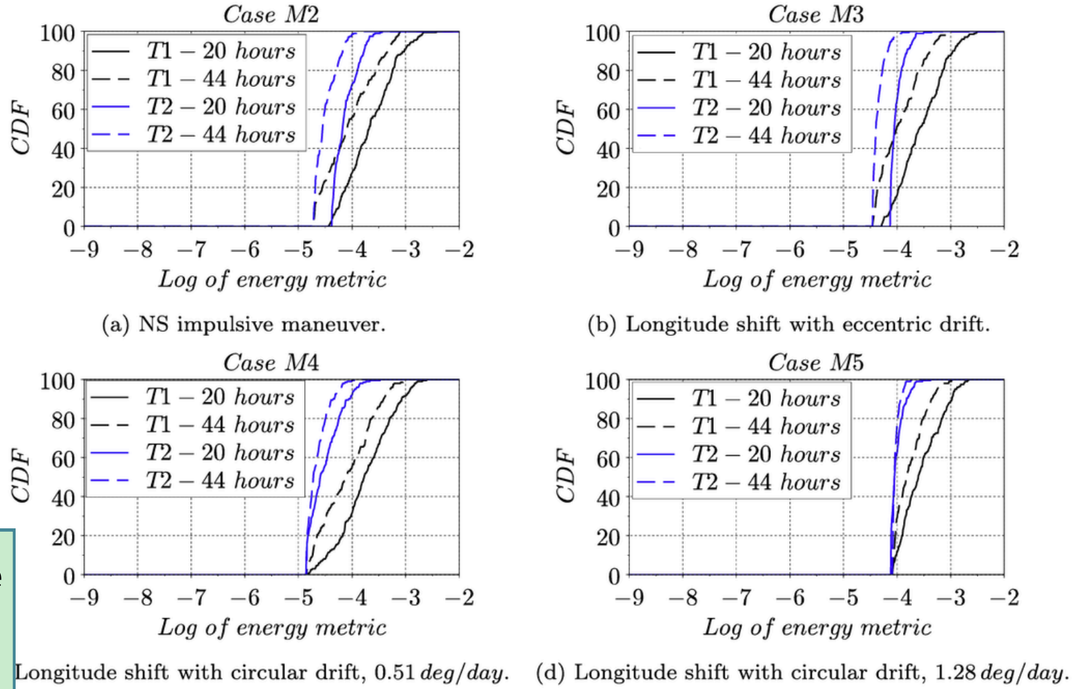
Epoch separation: Fixed ΔV spread over a window Δt.

Less spread of G2 Energy Metric for **20h** & **44h**.

Noise T1, T2, T3 having little to no effect on results.

M5 is flat because its apparent ΔV is not fixed.

Serra et. al. IVP Results



(a) NS impulsive maneuver. (b) Longitude shift with eccentric drift.

(c) Longitude shift with circular drift, 0.51 *deg/day*. (d) Longitude shift with circular drift, 1.28 *deg/day*.

Fig. 7. CDF of the logarithm of the energy metric for the *M2* to *M5* cases. Legend is the same as in Fig. 6.

Simulated Results [3]

Spread across 2-3 magnitudes of energy metric.
Between 1e-5 to 1e-2.

T1 and **T2** shows clear separation in results.

[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.

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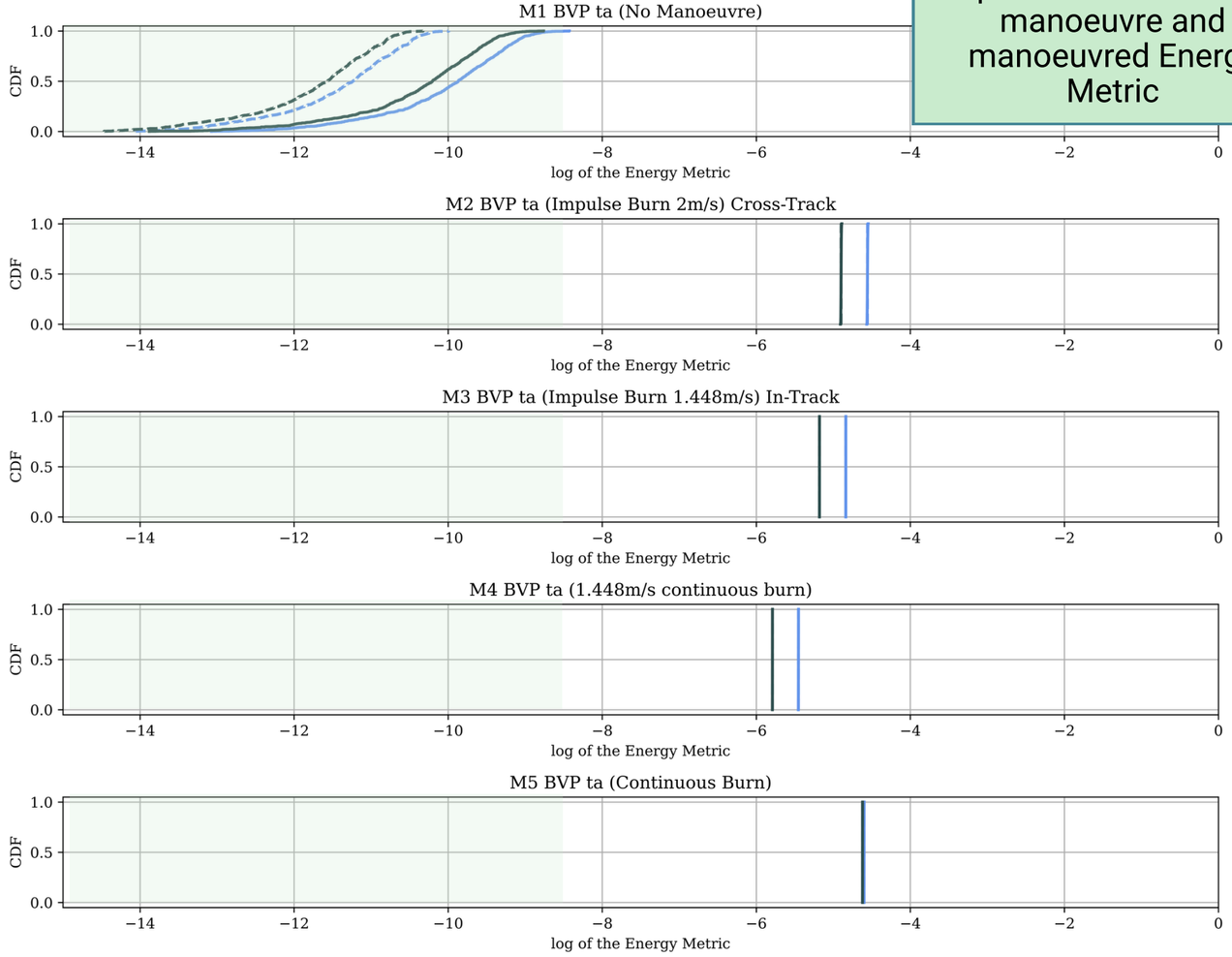
t_a Detecting Manoeuvres

Unable to seperate Energy Metric between M1 (no-manoeuvre) and M2-M5. Overlapping.

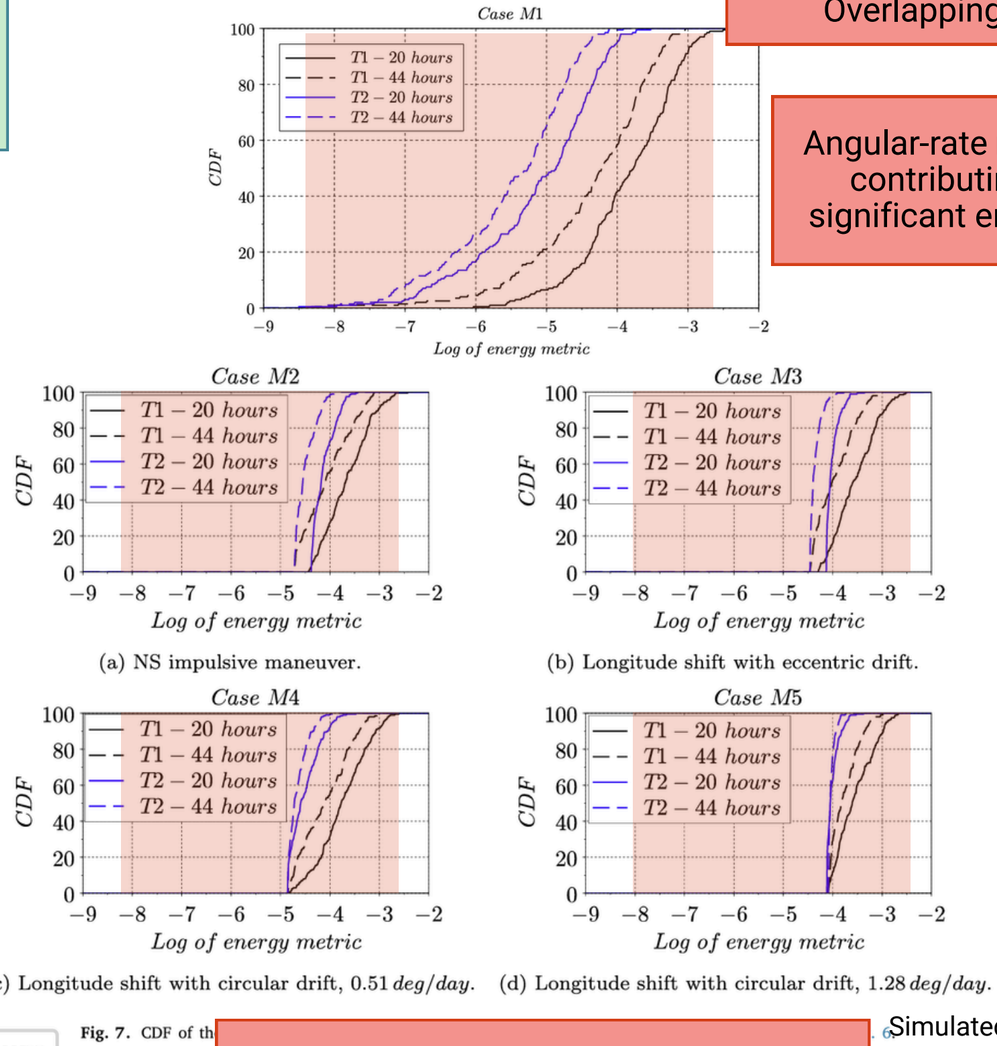
Angular-rate noise contributing significant errors.

Clear distinction and seperation between no-manoeuvre and manoeuvred Energy Metric

Our BVP Results



Serra et. al. IVP Results



(a) NS impulsive maneuver. (b) Longitude shift with eccentric drift.

(c) Longitude shift with circular drift, 0.51 *deg/day*. (d) Longitude shift with circular drift, 1.28 *deg/day*.

Fig. 7. CDF of th

Simulated Results [3]

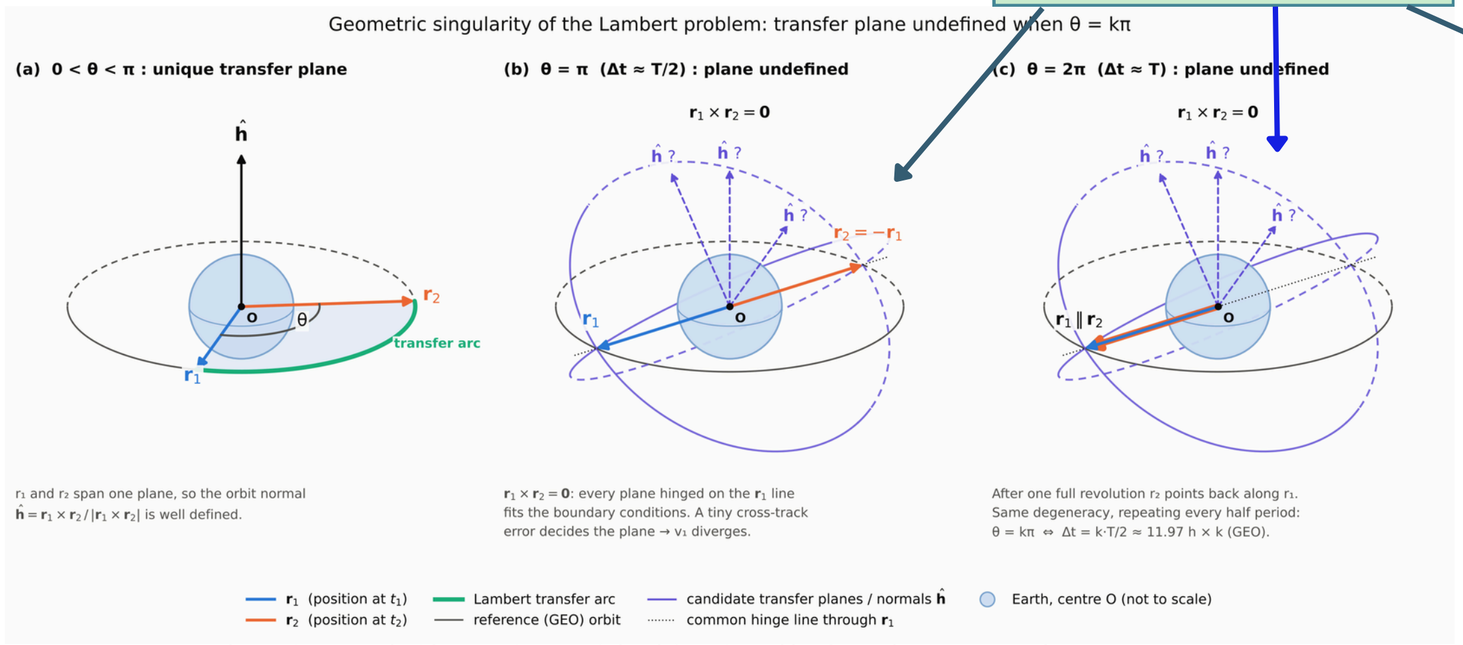
Empirically concluded that:
- Absolute no-thrusting threshold ~1e-6
- Unrealistic association above ~1e-2

[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.

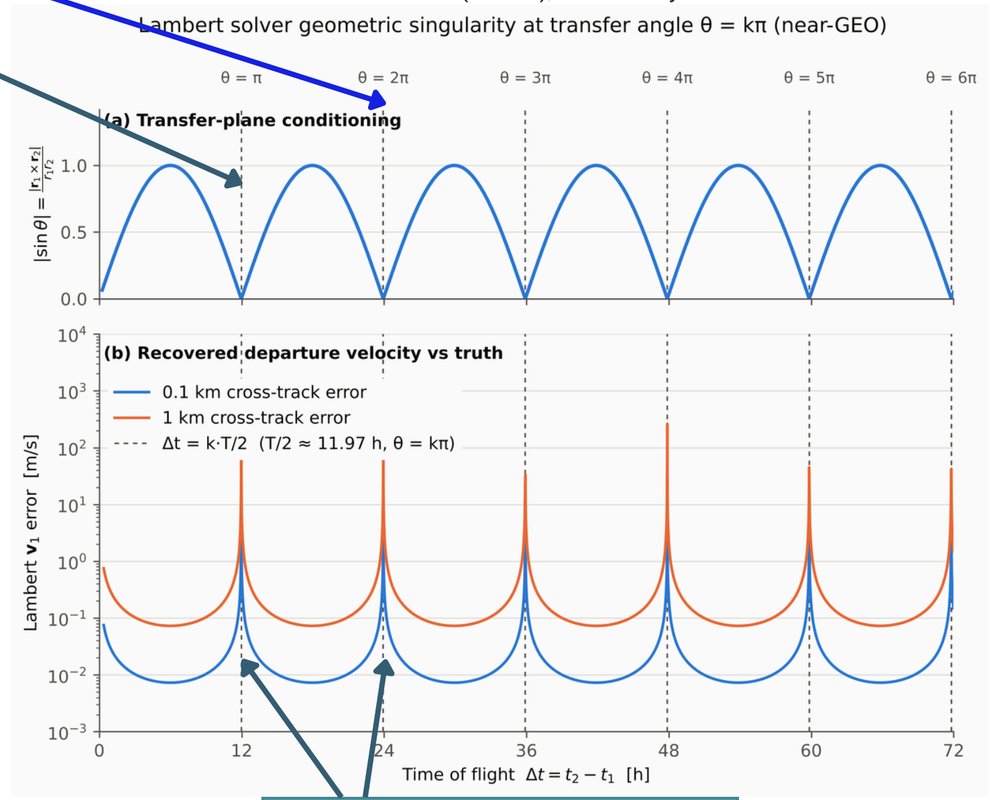
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Geometrical Singularity Limitation of Lambert Solver

r1 and r2 are collinear, transfer plane is undefined.



Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.



Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.

Transfer angle seen from Earth's centre:

$$\cos\theta = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{r_1 r_2}, \qquad |\sin\theta| = \frac{|\mathbf{r}_1 \times \mathbf{r}_2|}{r_1 r_2}$$

On a circular orbit, the transfer angle grows linearly with time.
With mean motion n:

$$\theta(\Delta t) = n\,\Delta t, \qquad n = \sqrt{\frac{\mu}{a^3}} = \frac{2\pi}{T}$$

Error in the recovered departure velocity v1. Spikes about 3-4 orders of magnitude.

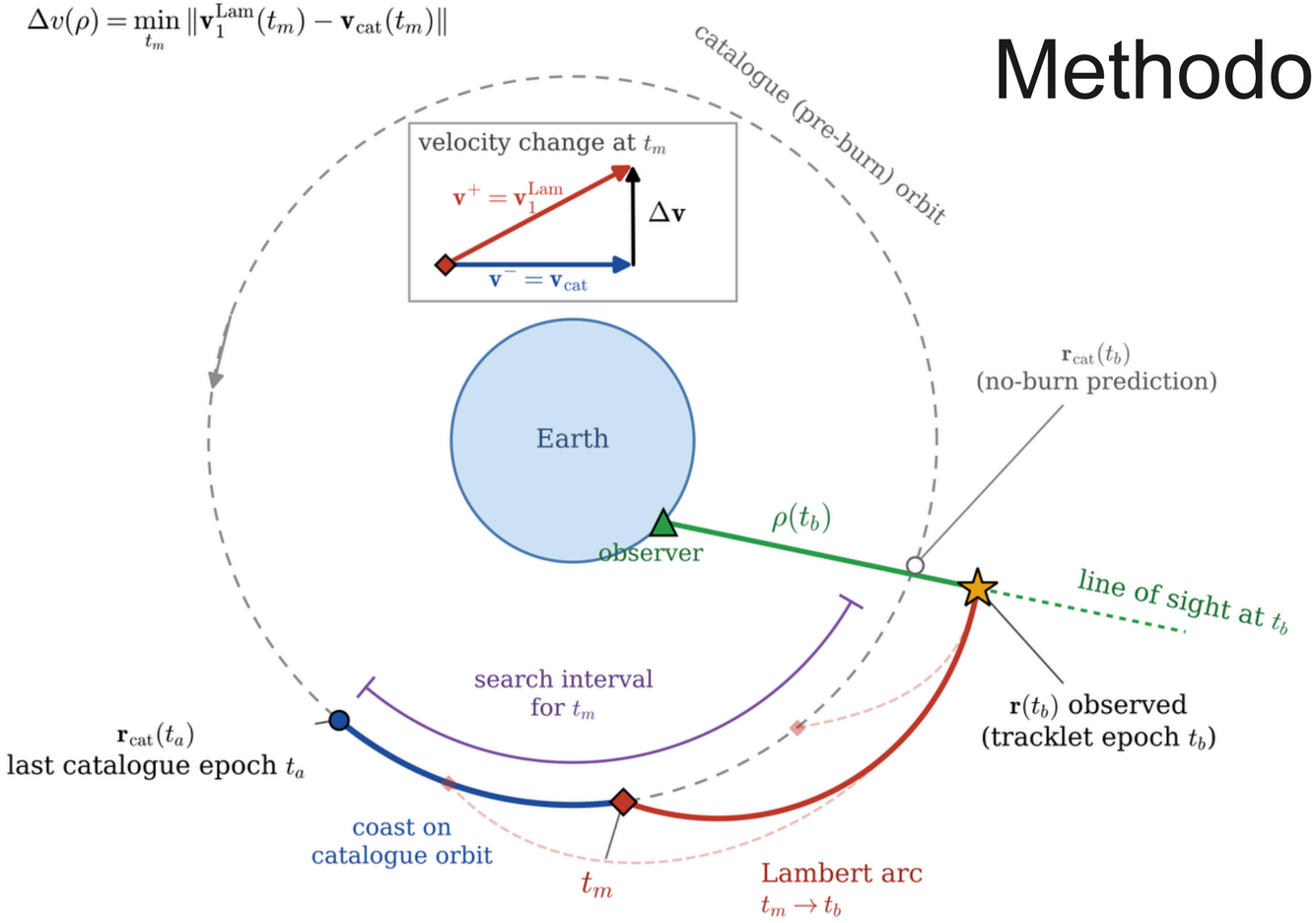
[6] Izzo, D. (2015). Revisiting Lambert's problem. Celestial Mechanics and Dynamical Astronomy, 121(1), 1–15. https://doi.org/10.1007/s10569-014-9587-y

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Impulsive Burn at Unknown t_m

Methodology

$$\Delta v(\rho) = \min_{t_m} \|\mathbf{v}_1^{\mathrm{Lam}}(t_m) - \mathbf{v}_{\mathrm{cat}}(t_m)\|$$



faint dashed: other candidate t_m (Δv minimised over t_m)

schematic, not to scale

$$\Delta v_{t_m}(\rho) = \min_{t_m \in [t_a,\, t_b]} \left\| \mathbf{v}_1^{\mathrm{Lam}} - \mathbf{v}_{\mathrm{cat}}(t_m) \right\|$$

$$G_2 = \frac{1}{2} \frac{|\Delta v_{t_m}(\rho)|^2}{TOF}$$

$$TOF = t_b - t_m$$

Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.

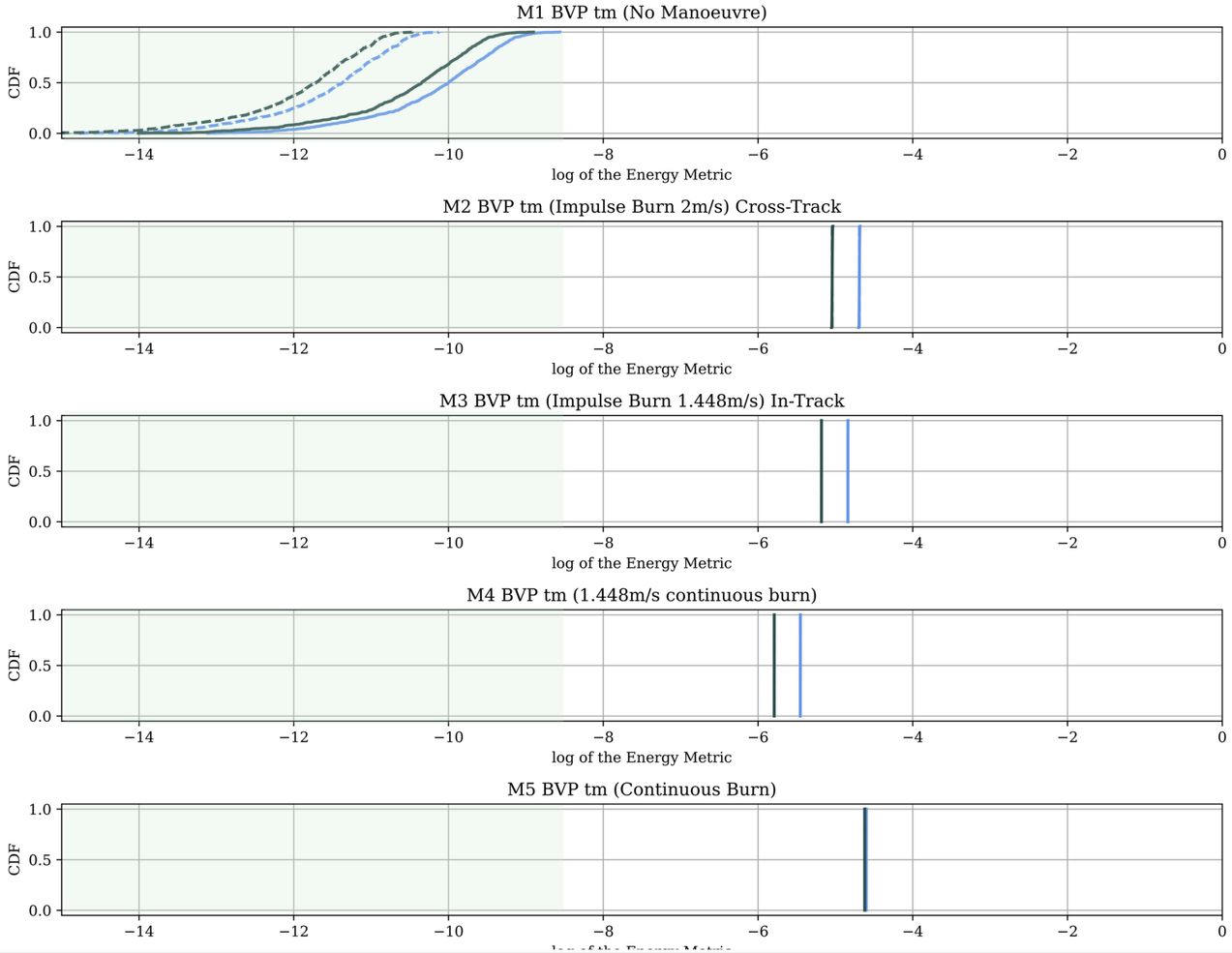
Detecting Manoeuvres t_m

Clear distinction and seperation between no-manoeuvre and manoeuvred Energy Metric

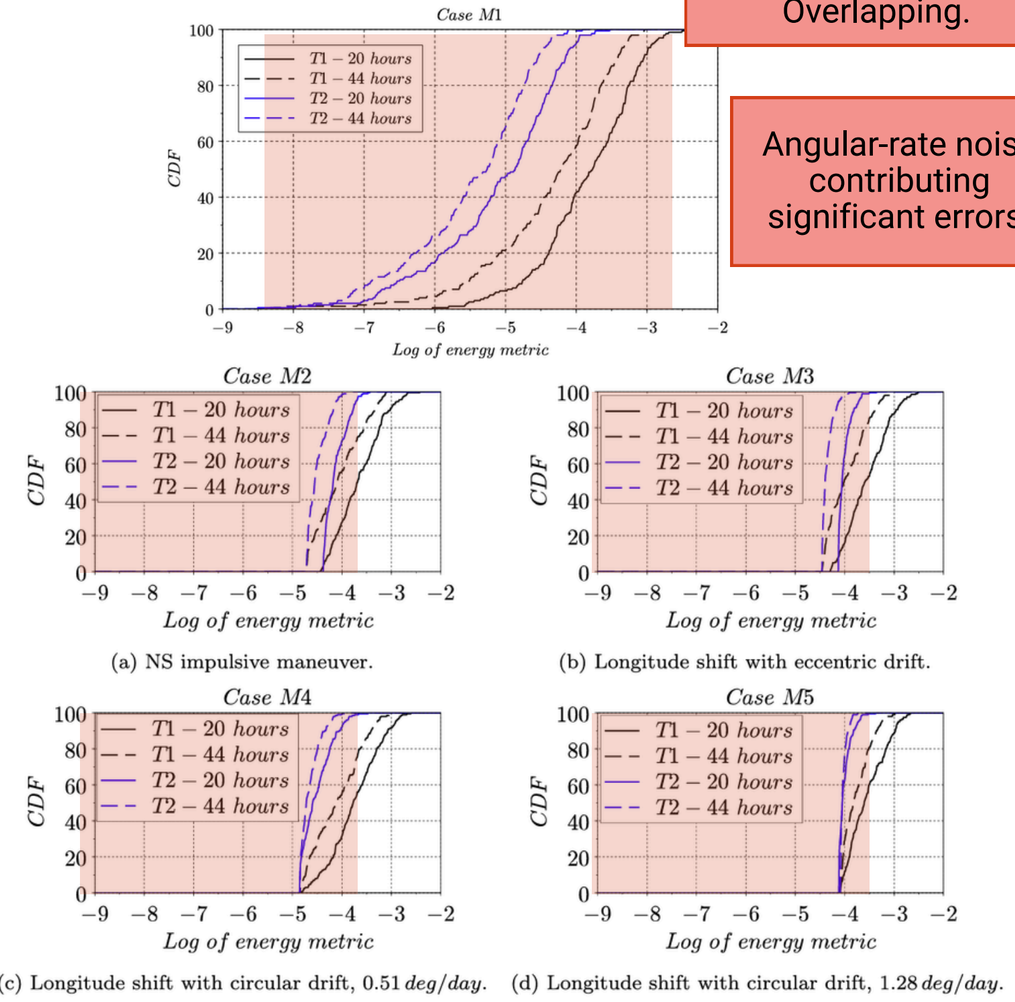
Unable to seperate Energy Metric between M1 (no-manoeuvre) and M2-M5. Overlapping.

Angular-rate noise contributing significant errors.

Our BVP Results



Serra et. al. IVP Results



(a) NS impulsive maneuver. (b) Longitude shift with eccentric drift.

(c) Longitude shift with circular drift, 0.51 *deg/day*. (d) Longitude shift with circular drift, 1.28 *deg/day*.

Fig. 7. CDF of the logarithm of the energy metric for the *M2* to *M5* cases. Legend is the same as in Fig. 6.

Simulated Results [3]

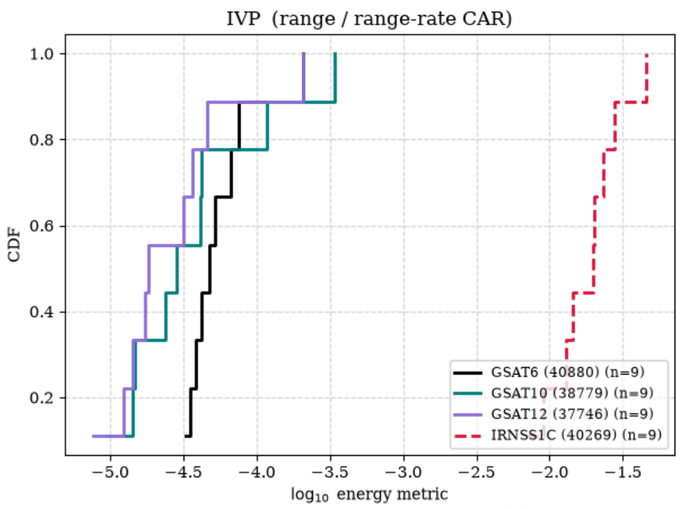
[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.

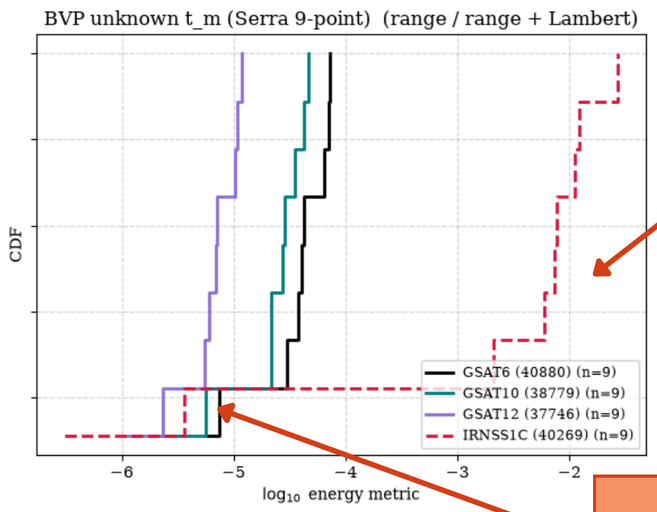
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TAROT Results t_m

GSAT6 (40880) (n=9)
GSAT10 (38779) (n=9)
GSAT12 (37746) (n=9)
IRNSS1C (40269) (n=9)

Real-data CDF per object: separation pulls IRNSS1C (dashed) right



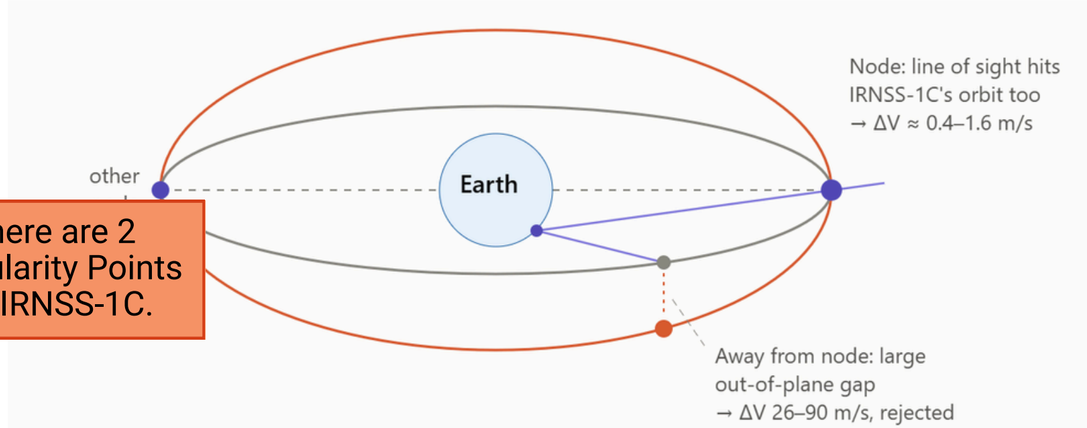


Recreated TAROT Results [3]

Just as Serra concluded from their IVP results [3].
Unable to separate the GSATs clusters.

Though we can reject the unplausible IRNSS-1C as the manoeuvred object.

There are 2 Singularity Points for IRNSS-1C.



At the node

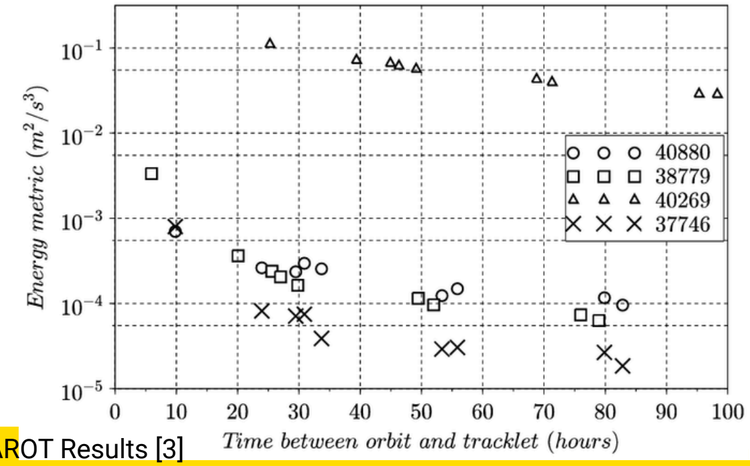
IRNSS-1C
GSAT

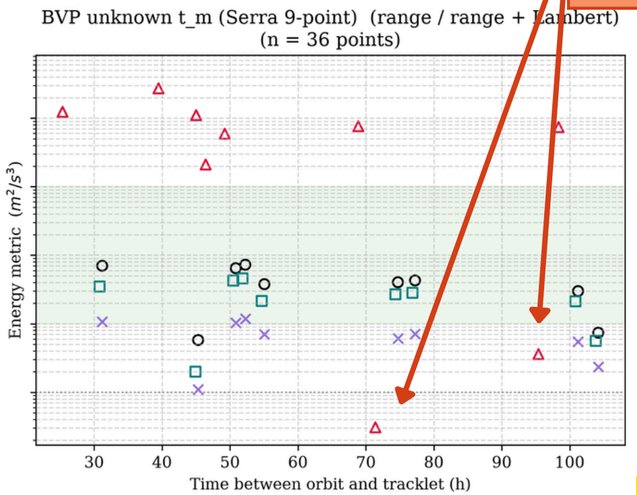
Same position, but velocities differ by v·sin(1.9°) ≈ 100 m/s out of plane.
The IVP sees this in the angle rates; the range/range BVP (position only) cannot.

GSAT / tracklet plane
Line of nodes (where planes cross)
Tracklet (observed GSAT position)
IRNSS-1C orbit (tilt exaggerated)
Line of sight from observer
Closest point on IRNSS-1C's orbit

Schematic created with AI assistance (Claude); reviewed by the author. Not to scale.

Serra et al. TAROT Results





TAROT Results [3]

GSAT6 (40880) GSAT10 (38779) GSAT12 (37746) IRNSS1C (40269)

[3] R. Serra, C. Yanez, and C. Frueh, 'Tracklet-to-orbit association for maneuvering space objects using optimal control theory', Acta Astronaut., vol. 181, pp. 271–281, Apr. 2021, doi: 10.1016/j.actaastro.2021.01.026.

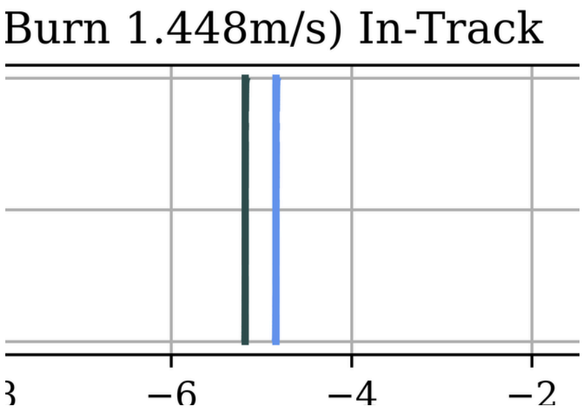
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Ongoing Work

Range/Range Continuous Energy-Optimal Control Theory	Recreating Serra et al.'s work with indirect shooting approach. Instead of implementing change in both boundary conditions (range/range) and Control Model (Impulse vs. Continuous), only pick one.
Testing with more epochs (e.g. 24h, 72h)	Increasing the simulated epochs tests, to test the known Geometric Singularity Limitation effect.
Comparative analysis and design to other maneouvre recovery work	Neither Serra et al. nor Seeto et al. implemented a manoeuvre recovery framework. Creating a comparison work of the ΔV in my work against others.
Further Real Data Sets Testing	Serra et al. stated that their real data test set is limited, especially noting that further work should include EWSK station tests. Our work should implement this test as well.

Conclusion

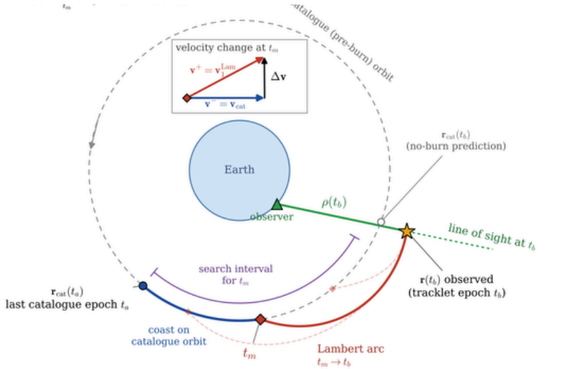
01



Manoeuvre Detection

Successfully detect maneouvre when comparing M1 to M2-M5.

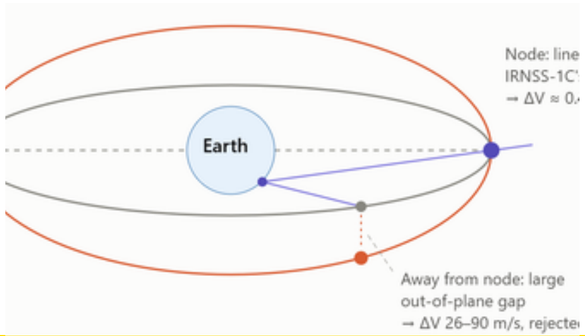
02



No dependence on Angle-rates

Less affected by Noise, limited to no spread of energy metric depending on noise.

03



Limited by Plane Geometry

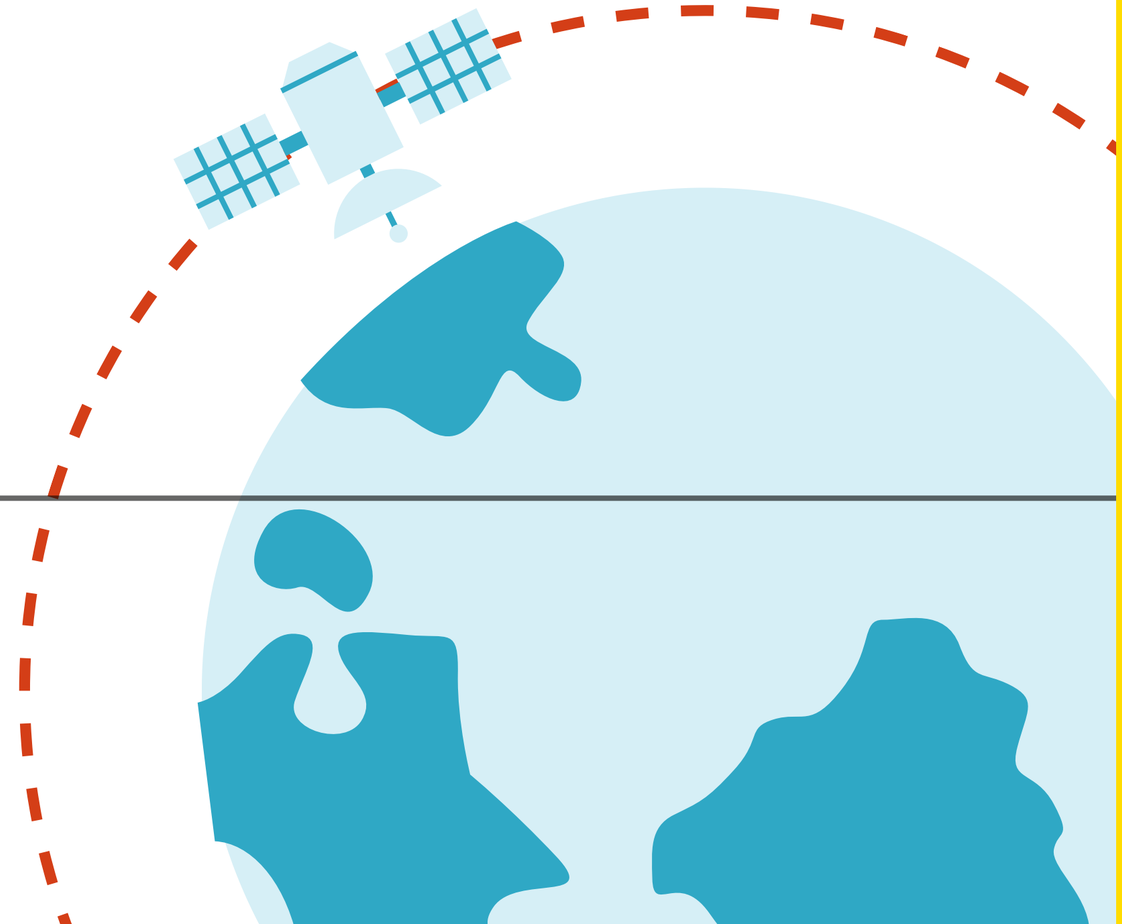
Plane-crossing coincidence, line-of-sight passes close to IRNSS-1C's orbit.

Schematics created with AI assistance (Claude); reviewed by the author. Not to scale.

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Thank you

Any Questions?



Honours Thesis

Supervised by Dr Yang Yang
Written by Angela Ng



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